Dynamic Models of Fixed Capital Stocks and Their Application in Industrial Ecology

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Keywords:

consequential LCA
dynamic input-output analysis
dynamic stock model
industrial metabolism
material flow analysis (MFA)
perpetual inventory method

Supporting information is available on the *JIE* Web site

Summary

Industrial assets or fixed capital stocks are at the core of the transition to a low-carbon economy. They represent substantial accumulations of capital, bulk materials, and critical metals. Their lifetime determines the potential for material recycling and how fast they can be replaced by new, more efficient facilities. Their efficiency determines the coupling between useful output and energy and material throughput. A sound understanding of the economic and physical properties of fixed capital stocks is essential to anticipating the long-term environmental and economic consequences of the new energy future. We identify substantial overlap in the way stocks are modeled in national accounting, dynamic material flow analysis, dynamic input-output (I/O) analysis, and life cycle assessment (LCA) and we merge these concepts into a common framework for modeling fixed capital stocks. We demonstrate the usefulness of the framework for simultaneous accounting of capital and material stocks and for consequential LCA. We apply the framework to design a demand-driven dynamic I/O model with dynamic capital stocks, and we synthesize both the marginal and attributional matrix of technical coefficients (A-matrix) from detailed process inventories of fixed assets of different age cohorts and technologies. The stock modeling framework allows researchers to identify and exploit synergies between different model families under the umbrella of socioeconomic metabolism.

Introduction

Fixed Capital Stocks and the Transition to a New Energy Future

The transition to a sustainable energy future (IEA 2010) is shaped by many factors, such as the availability and timing of new, more efficient technologies (IEA 2010), potential scarcity of mineral resources (Graedel et al. 2012), and the speed at which existing assets can be replaced (Davis et al. 2010). A central and novel challenge is that a multitude of new or improved technologies, which were developed and tested on a small scale, need to replace existing assets over a long time, in all world regions, and on a large scale. To understand how to mitigate climate change and promote human development at the same time requires models that integrate physical and technological aspects of new energy technologies with economic and social aspects of the distribution of their benefit within society. Industrial ecology (IE) principles and the concept of socioeconomic or anthropogenic metabolism (Baccini and Brunner 1991; Fischer-Kowalski and Weisz 1999) represent a framework for this integration. An important example of model integration within that framework is the combination of environmentally extended input-output (I/O) analysis (IOA) and material flow analysis (MFA) to better understand the coupling between the physical and monetary layer of industrial systems (Duchin 1992, 2009; Kytzia et al. 2004).

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DOI: 10.1111/jiec.12149

Editor managing review: Sangwon Suh

Volume 19, Number 1

Important examples of combining IOA and MFA include the waste I/O (WIO) model (Kondo and Nakamura 2002), WIO-MFA (Nakajima and Nakamura 2006), economically extended MFA (Kytzia et al. 2004), different versions and amendments of physical and monetary I/O analysis (Duchin 2009; Weisz and Duchin 2006; Hubacek and Giljum 2003; Dietzenbacher et al. 2009; Giljum and Hubacek 2009; Wood et al. 2009), hybrid supply-and-use tables (Schmidt and colleagues 2010), and IOA with mixed units (Hawkins et al. 2007). All these approaches are static models; they represent snapshots of the industrial metabolism recorded over a certain accounting period, typically 1 year. They cover the *flows* of materials and energy through industrial processes and their distribution among industry and end users, but not the *stocks* or *fixed assets*, such as production equipment, buildings, infrastructure, or vehicles. Pauliuk and Müller (2014) compile a systematic overview of the role of stocks in socioeconomic metabolism. They identify four properties of industrial assets, or fixed capital stocks, that need to be understood when developing strategies for a new energy future:

Capital containers and *resource repositories*: Low-carbon energy technologies are capital and material intensive and the impact of building and maintaining these installations can be much higher than the impact associated with their use (Frischknecht et al. 2007).

Dynamics determiners: The service lifetime of industrial installations determines how quickly they can be replaced and thus how quickly new energy technologies can penetrate (Davis et al. 2010). The development of in-use stocks over time determines future resource demand and the potential for material recycling (Vidal et al. 2013; Davis et al. 2010).

Consumption couplers: The I/O structure of an industrial sector is determined by the respective "production recipes" of all the factories and installations it comprises. The turnover and aging of the productive capital stock determines how the input structure of the sector changes over time (Davis et al. 2010).

State of the Art of Dynamic Capital and Material Stock Modeling in National Accounting, Material Flow Analysis, and Input-Output Analysis

Dynamic modeling of capital stocks has a long tradition in economic accounting, and we refer to the Organization for Economic and Cooperative Development (OECD 2009) for an overview. Many statistical offices apply a dynamic capital stock model called the perpetual inventory method (PIM), which tracks the different age cohorts of capital investment over time and determines the retirement of capital assets in different economic sectors according to their physical service lifetimes (OECD 2001, 2009). Here, fixed capital assets are seen as capital containers. On the physical side, Baccini and Bader (1996), van der Voet and colleagues (2002), and Müller (2006) describe and apply dynamic models of material stocks (resource repositories) using age cohorts. Müller (2006) introduces the stock-driven model, where inflows and outflows of products to build up and maintain in-use stocks are determined from exogenous time series for total stock size and the service lifetime of the different age cohorts (in-use stocks as dynamics determiners). Dynamic models of stocks as consumption couplers are state of the art in models of the vehicle fleet (Wang 1999; Melaina and Webster 2011; Pauliuk et al. 2012) and the building stock (Kohler 2006; Sandberg and Brattebø 2012; Pauliuk et al. 2013).

In dynamic I/O analysis, investments into capital stocks or research and development are considered a prerequisite for expanding production capacity or increasing factor productivity. The capital stock itself, as well as its dynamics and impact on technical coefficients, are often not considered, however. Examples include Leontief (1953), Vogt and colleagues (1975), Lange (1980), ten Raa (1986), Los (2001), and Hoekstra and Janssen (2006). Duchin and Szyld (1985) propose a generalization of the model used by Leontief (1953), which ensures the existence of a solution with positive industry output by using exogenous capacity estimates based on historic trends. Leontief and Duchin (1986) apply this model to estimate the impact of future labor automation on the A-matrix of the U.S. for 1990 and 2000. To derive the change of the A-matrix over time, they do not use a dynamic stock model of productive assets of different age cohorts, as first pointed out by Carter (1963). Instead, they estimate the future technical coefficients by directly modifying them according to assumptions on future productivity gains. The model developed by Duchin and Szyld (1985) neglects dismantling and demolishing, which are crucial when modeling a replacement of industrial stocks during a metabolic transition. The distinction between maintenance (applied to existing facilities) and replacement (new assets that replace old ones) is not made. The model of Duchin and Szyld (1985) inspired other scholars: Pan (2006) and Ryaboshlyk (2006) adjust the technical coefficients in their model according to the share of new and existing industries within a sector. Pan (2006) applies rates for depreciation and obsolescence of assets to the capital stock to model how it ages with time and allows assets to be idle to avoid overproduction in times of stalling demand. Pan's model does not differentiate between different age cohorts; depreciation, obsolescence, and idleness rates are applied to the entire capital stock as a unit. This capital stock model resembles what is called a "leaching model" in dynamic MFA (van der Voet et al. 2002). Idenburg and Wilting (2000) consider age cohorts and fixed economic lifetime to model retirement of existing assets in their version of the Duchin and Szyld (1985) model. They calculate the overall A-matrix as synthesis of the A-matrices of the different age cohorts. Lennox and colleagues (2005) specify the lifetime distribution of industrial assets to model their turnover and they consider possible underutilization of productive capacity. They use inventory data for the material intensity of specific energy conversion technologies to determine the total material demand from new installations. A detailed discussion of all approaches is given in the Supporting Information on the Web.

The review shows two things: (1) The stock models used in capital accounting, dynamic MFA, and dynamic IOA seem to converge toward an age-cohort–specific description of technological or economic attributes of fixed assets. (2) There is no dynamic modeling framework, however, that includes both capital *and* material stock dynamics, that includes all life cycle stages of industrial assets, and that consistently distinguishes between different age cohorts of industrial assets or technology.

Scope and Research Questions

We believe that better exploitation of synergies between different model families under the umbrella of socioeconomic metabolism may allow researchers to work more efficiently and may increase the validity and visibility of the model results. In this article, we develop a harmonized accounting and modeling framework to study industrial capital stocks in their role as capital containers, resource repositories, dynamics determiners, and consumption couplers. This framework can improve models that assess the economic and physical aspects of the transition to a sustainable energy future.

The article has four parts:

1. We establish a connection between dynamic modeling of capital stocks (PIM) and material stocks (dynamic MFA).
2. We propose a generic way of modeling the buildup, maintenance, and dismantling of industrial assets.
3. We derive a general dynamic I/O model based on our framework for accounting capital stocks and investment flows and discuss its potential application in IE.
4. The general framework of fixed capital stocks can provide useful insights into other fields of research, and we provide two examples: consequential life cycle assessment (CLCA) and potential synergies between IE and the integrated assessment modeling community.

Dynamic Modeling of Economic and Physical Aspects of the Fixed Capital Stock

First, we list all indices and system variables at their maximum level of specification (table 1). The meaning of the variables is explained when they appear in the text for the first time. By summing up over indices, one obtains more aggregated arrays, for example, $C(t,i,J) = \sum_{t' \leq t} C(t,t',i,J)$. This summation is not always made explicit in the text.

In economic accounting, two types of fixed capital stock are distinguished (figure 1, right). The *gross capital stock* C is the sum of all historic investments into assets that are still in operation at the time of measurement, revalued at current purchasers prices of equivalent new capital goods (OECD 2009). The gross fixed capital stock C in an economy at time *t* consists of goods or products *i* that can be specified by the industrial sector *J* where they reside, their year of production *t'*, and their physical lifetime τ_C in that industry (equation (1)):

$$\begin{aligned} C &= C(t,t',i,J) \\ \tau_C &= \tau_C(t',i,J) \end{aligned} \qquad (1)$$

The *net capital stock* comprises the same productive assets, but determines their value based on the current market value for the assets of different age cohorts (OECD 2001, 2009). The fixed capital stock contributes to the production process; this contribution is called capital service (OECD 2009). It has both a value and a volume aspect.

The value aspect is modeled by the consumption of fixed capital, *CFC*, which is the diminishment of the net capital stock of existing assets between 2 consecutive years. The consumption of fixed capital does not have a direct physical counterpart because it relies on assumptions on future revenue, discount rates, or depreciation schemes.

The volume aspect or physical aspect of an accumulation of fixed capital in a certain sector can be represented by its production capacity G. This capacity is a property of industrial installations, which, in turn, consist of different components and products. The lifetime τ_G of an installation differs from the physical lifetimes τ_C of the capital goods it is made of, because the different components of the installation, such as buildings, machines, or control equipment, are maintained or replaced at different intervals. Moreover, the lifetime τ_A of the net capital goods in the asset is derived from accounting principles, and it can differ from their physical lifetime τ_C (OECD 2009).

To build up or maintain production capacity, a certain fraction of a country's final output *y* needs to be invested into industrial capital each year. This investment flow, together with investments into dwellings, is termed *gross fixed capital formation* (*GFCF*) (European Commission 2008). The *GFCF* specifies the products that are invested, but not the industrial sector in which the investment occurs. A more specific breakdown of investments by target industrial sector *J* is represented by the investment matrix *K(t,i,J)* (European Commission 2008) (figure 1, left). The flows of goods in *K* are invested to build up, maintain, or demolish production capacity. A general breakdown of *K* therefore consists of three parts: K_B is associated with constructing new assets; K_R with maintaining existing assets; and K_D with dismantling and demolishing obsolete or retired assets (equation (2)).

$$K = K_B + K_R + K_D \qquad (2)$$

The lifetime distribution function $\lambda_C(t,t',i,J)$ denotes the probability that the asset of product *i* in industry *J* acquired in year *t'* leaves the fixed capital stock in year *t* with an age of *t-t'*. When the different age cohorts of capital investment are recorded, the gross capital stock $C(t,i,J)$ can be calculated from deflated past investments and λ_C (equation (3)). The term in brackets denotes the fraction of the original investment that is still part of the fixed capital stock. This dynamic model of the capital stock is called PIM (European Commission 2008; OECD 2009).

$$C(t,i,J) = \sum_{t' \leq t} K(t',i,J) \cdot \left(1 - \sum_{t' \leq t'' \leq t} \lambda_C(t'',t',i,J)\right) \cdot T \qquad (3)$$

Table 1 Overview of the variables used in the system, their symbols, and respective units

Index name and description	*Symbol*	*Domain*
Model time	t	Case specific
Age-cohort or vintage	t'	Same as model time
Product or commodity category	$i,j,k,\ \ldots$	$1 \ldots N_{Prod}$
Industry category or sector	$I,J,K,\ \ldots$	$1 \ldots N_{Ind}$
Material type (good or substance)	m	$1 \ldots N_{Mat}$
Name and description, general variables	**Symbol and dependency**	**Unit**
Accounting period and discrete model increment: 1 year	T	yr
Total output, by commodity or by industry, from I/O model	$x(t,i)$, $x(t,J,t')$	\$/yr
Final demand, exogenous	$y(t,i)$	\$/yr
Final demand without investments in industrial assets, exogenous	$\tilde{y}(t,i)$	\$/yr
Gross capital stock (year, age-cohort, product, industry)	$C(t,t',i,J)$	\$
Remaining original production capacity (year, age-cohort, industry)	$G_0(t,t',J)$	\$/yr
Effective (nominal) production capacity, measure of capital service	$G(t,t',J)$	\$/yr
New production capacity in sector *J* installed in year t'	$G_{in}(t',J)$	\$/yr
Production capacity in sector *J* built in year t' and retiring in year t	$G_{out}(t,t',J)$	\$/yr
Gross fixed capital formation	$GFCF(t,i)$	\$/yr
Investment matrix (GFCF broken down by target sector)	$K(t,i,J)$	\$/yr
Investment matrix for building up new capacity	$K_B(t,i,J)$	\$/yr
Investment matrix for maintaining existing capacity	$K_R(t,i,J)$	\$/yr
Investment matrix for disposing of retiring capacity	$K_D(t,i,J)$	\$/yr
Lifetime and probability distribution of retirement of capacity	$\tau_G(t',J)$, $\lambda_G(t,t',J)$	yr, 1
Lifetime and probability distribution of retirement of gross capital	$\tau_C(t',i,J)$, $\lambda_C(t,t',i,J)$	yr, 1
Lifetime and probability distribution of retirement of net capital	$\tau_A(t',i,J)$, $\lambda_A(t,t',i,J)$	yr, 1
Lifetime and probability distribution of discard of materials in industries	τ_M, $\lambda_M(t,t',i,J,m)$	yr, 1
Age-efficiency (factor that relates effective to nominal capacity)	$\eta_1(t,t',J)$	1
Utilization rate (factor that relates output to effective capacity)	$\eta_2(t,t',J)$	1
Material concentration of products	$\mu(t',i,m)$	kg/\$
Material inflow to industrial capital stocks	$M_{in}(t',i,J,m)$	kg/yr
Material outflow from industrial capital stocks	$M_{out}(t,i,J,m)$	kg/yr
Material stock in fixed capital of product *i* in industry *J*	$M(t',i,J,m)$	kg
Matrix of specific requirements for building up average capacity	$B^a(t',i,J)$	\$/(\$/yr)
Matrix of specific requirements for building up new capacity	$B^n(t',i,J)$	\$/\$
Matrix of specific requirements for maintaining existing capacity	$R(t,i,J)$	\$/\$
Matrix of specific requirements for disposing of retiring capacity of year *t'*	$D(t',i,J)$	\$/\$
Name and description, variables for I/O model		
Square matrix of allocated technical coefficients, average	$A(t,i,J)$, $A^a(t,i,J)$	\$/\$
Square matrix of allocated technical coefficients for installations of a specific age-cohort	$a(t',i,J)$	\$/\$

Note: Dollars (\$) represent the monetary unit, but any other currency can be used as well. All monetary values are recorded in constant prices. I/O = input-output; GFCF = gross fixed capital formation; yr = year; kg = kilograms.

A similar model can be set up for the production capacity G, where G_{in} denotes additions to capacity, G_{out} retirements of existing capacity, and λ_G the probability distribution of capacity retirement (equation (4)). The process efficiency of an asset may change during its operational life; it may require more maintenance and have therefore more idle days as it gets older or it may be upgraded by installing new process control equipment. To model these effects, an empirical age efficiency η_1 is assigned to each asset in stock. It is the ratio of the current physical service to the original service provided by the asset (OECD 2009). If we use production capacity to measure the physical service provided by an asset, we can use the age-efficiency parameter to determine the current or effective production capacity G that results from the original installations G_{in} (equation (4)). Here, G_0 is the remaining original production capacity that is still in use.

$$G_{out}(t,J) = \sum_{t' \le t} G_{in}(t',J) \cdot \lambda_G(t,t',J)$$
$$G(t,t',J) = \eta_1(t,t',J) \cdot G_{in}(t',J) \cdot \left(1 - \sum_{t' \le t'' \le t} \lambda_G(t'',t',J)\right) = \eta_1(t,t',J) \cdot G_0(t,t',J) \qquad (4)$$

The estimation of asset or product lifetimes and lifetime distributions has a long tradition in economic accounting (Winfrey 1935; Lennox et al. 2005; OECD 2009) and within the

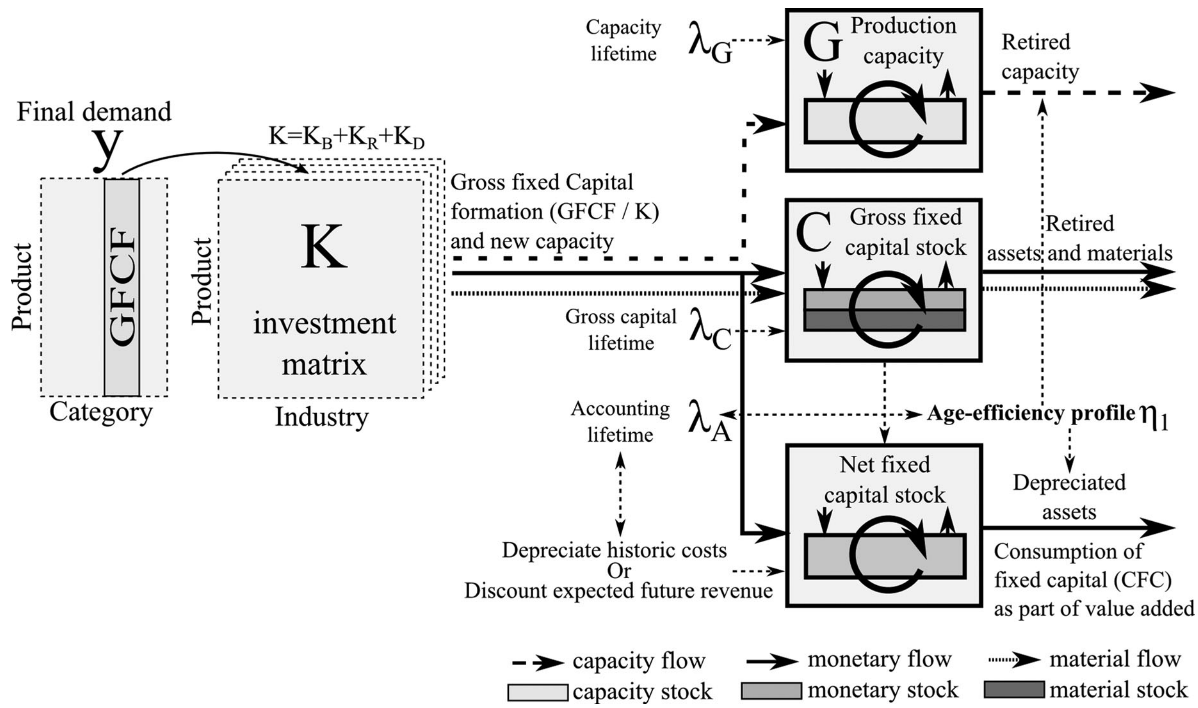

Figure 1 Left: Gross fixed capital formation and its disaggregation into the investment matrix *K*. Right: schematic drawing of the perpetual inventory method. Production capacity G, gross fixed capital stock C, and the net capital stock are three different concepts to quantify the same industrial assets according to the service they provide, their gross economic value, and their net economic value.

social metabolism community (Müller et al. 2007; Murakami et al. 2010; Oguchi et al. 2010; Kagawa et al. 2011). Product lifetime distributions λ_M are commonly used in age-cohort–based dynamic material stock models to estimate the accumulated material stock M and the future scrap flows M_{out} that result from historic material consumption M_{in} (van der Voet et al. 2002; Müller et al. 2007; Baccini and Bader 1996; Müller 2006) (equation (5)).

$$\begin{aligned} M_{out}(t,i,J,m) &= \sum_{t'\le t} M_{in}(t',i,J,m)\cdot\lambda_M(t,t',i,J,m) \\ M(t,i,J,m) &= \sum_{t'\le t} M_{in}(t',i,J,m)\cdot \\ &\quad \left(1-\sum_{t'\le t''\le t}\lambda_M(t'',t',i,J,m)\right)\cdot T \end{aligned} \tag{5}$$

By comparing equations (3) and (5), we see that the age-cohort–based capital and material stock models are technically identical. One represents the monetary and the other the material layer of capital stocks and investment flows. We can establish a formal identity between the perpetual inventory method and the age-cohort–based material stock model by considering that the physical asset lifetime, that is, the time span the asset remains in stock before it is demolished, equals the useful lifetime of the materials in that asset. It is also important to note that the physical asset lifetime τ_C may differ from their accounting or depreciation lifetime τ_A, because the latter is derived from accounting principles and not necessarily from the actual physical lifespan (OECD 2009). For each product *j* in each industry *J*, we introduce the array of average material content $\mu(t',i,m)$ of material *m* per dollar of commodity *i* in year *t'*. The identity between the two models can be used to determine the total material stock M contained in the fixed capital stock from data on historic investment (equation (6)):

$$\begin{aligned} &\lambda_M(t,t',i,J,m) \equiv \lambda_C(t,t',i,J) \\ &M_{in}(t',i,J,m) = \mu(t',i,m)\cdot K(t',i,J) \\ &M(t,i,J,m) = \sum_{t'\le t} M_{in}(t',i,J,m)\cdot \\ &\qquad \left(1-\sum_{t'\le t''\le t}\lambda_M(t'',t',i,J,m)\right)\cdot T \\ &M(t,i,J,m) = \sum_{t'\le t}\mu(t',i,m)\cdot K(t',i,J)\cdot \\ &\qquad \left(1-\sum_{t'\le t''\le t}\lambda_C(t'',t',i,J)\right)\cdot T \\ &M(t,i,J,m) = \sum_{t'\le t}\mu(t',i,m)\cdot C(t,t',i,J) \end{aligned} \tag{6}$$

The identity between the two stock models opens up the opportunity to cover both physical and monetary aspects of industrial assets within a common modeling framework. Parallel accounting of both aspects would allow us to identify the contribution of industrial assets to overall material demand and in-use stocks of materials, as well as to determine the future potential for material recycling from obsolete assets. It would allow us to break down the material stocks in industrial assets by industrial sector, which would provide a much higher level of detail than is currently available from statistics about the end use of different materials.

Modeling the Buildup, Dismantling, and Maintenance of Fixed Capital and Production Capacity

We propose to measure the physical service provided by an industrial asset in terms of its production capacity G, and inspired by Lennox and colleagues (2005), we introduce the capacity utilization rate $\eta_2(t,t',J)$, which relates industrial output x to production capacity (equation (7)):

$$\begin{aligned} x(t,J) &= \sum_{t'} \eta_2(t,t',J)\cdot G(t,t',J) \\ &= \sum_{t'} \eta_2(t,t',J)\cdot \eta_1(t,t',J)\cdot G_0(t,t',J) \\ &= \sum_{t'} \eta_2(t,t',J)\cdot \eta_1(t,t',J)\cdot G_{in}(t',J)\cdot \\ &\quad \left(1-\sum_{t'\le t''\le t} \lambda_G(t'',t',J)\right) \end{aligned} \tag{7}$$

The upper plot in figure 2 illustrates the relation between output, effective capacity, and originally installed capacity according to equation (7) for the special case where all assets are decommissioned after τ_G. The life cycle of an industrial asset consists of the three stages, buildup, operation/maintenance, and dismantling, and each stage can be characterized by a matrix of specific capital requirement per unit of capacity. The matrix symbols are B^n for buildup, R for maintenance or replacement, and D for dismantling, and the row index of each matrix indicates the capital good i and the column indicates the target sector J (figure 2, middle plot).[1]

There are several ways of obtaining these matrices:

(1) The *average* capital requirements matrix B^a is determined as the amount of capital stock C per unit of output x (Miller and Blair 2009) (equation (8)[2]:

$$B^a = C\cdot \hat{x}^{-1} \tag{8}$$

This formula treats the entire capital stock as one unit and does not distinguish between different age cohorts. B^a is measured in \$/(\$/year) and not in \$/\$ as the other capital intensity matrices defined above, because it relates to the capital stock and not to the flow of new capacity into the stock.

(2) In the marginal approach (Rose 1984; Fleissner et al. 1993), the capital intensity of new production capacity is determined by dividing the investment into new assets K_B by the resulting capacity addition G_{in} (equation (9)). We use an analog approach to connect maintenance investments R to the total gross production capacity G_0 and demolition investments D to the retiring capacity flow G_{out} (equation (9)).

$$\begin{cases} B^n = K_B\cdot \hat{G}_{in}^{-1} \\ R = K_R\cdot \hat{G}_0^{-1} \\ D = K_D\cdot \hat{G}_{out}^{-1} \end{cases} \tag{9}$$

In the I/O literature, B^n is called "matrix of marginal capital coefficients" (Rose 1984), but we avoid this term because it may be confused with the concept of marginality used in neoclassical economics or in CLCA, which refers to marginal processes that are selected by market mechanisms and not by which age cohort they belong to.

(3) A comprehensive set of life cycle inventories (LCIs), that describes existing and future technologies and that covers construction, operation, and the end-of-life phase of different assets, would allow us to determine the matrices B^n, R, and D by converting the physical inventories to monetary flows using appropriate price data. This approach would yield much detail about physical inputs, especially for different materials, but might overlook ancillary inputs, such as planning, insurance, or other fixed costs with no material counterpart. To overcome the limited scope of physical inventories, I/O models can be used to estimate additional service inputs. This type of hybridization has some tradition in the research community (Suh et al. 2004; Stromman et al. 2009; de Haes et al. 2004; Lenzen and Crawford 2009). Using LCIs of very specific or future technologies, such as electricity generation with carbon capture and storage, may help to increase the resolution of different industrial sectors and may amend the predictive capacity of the model (Hertwich et al. 2014; Gibon et al. 2014). LCIs can also be used to determine age-efficiency profiles $\eta_1(t,t',J)$.

The production recipe of the industrial processes that belong to an age cohort t' is described by their respective technical coefficients $a(t',i,J)$, which denote the amount of commodity i required to produce a unit of output J. Multiplication with output x yields the interindustry flows into a specific age cohort of assets $a(t',i,J)\cdot x(t,J,t')$ (figure 2, lower plot). In the next section, we will synthesize the industry-wide A-matrix from the inventories of individual age cohorts of industrial processes. For a systematic overview of how to model changes in technical coefficients, we refer to Rose (1984).

Modeling the turnover of industrial assets is a difficult task, because the different parts or materials that constitute an asset may have different lifetimes and because some parts may be replaced or maintained more frequently than others. An oil refinery, for example, consists of infrastructure such as roads or pipes, different reactors, electronic equipment, or catalysts, which all have their specific material content and lifetimes. Installation of new process control equipment or catalysts may significantly increase capacity, without increasing infrastructure stocks. The general relation between capital investment and capacity G is therefore more complicated than equation (9) suggests. Because

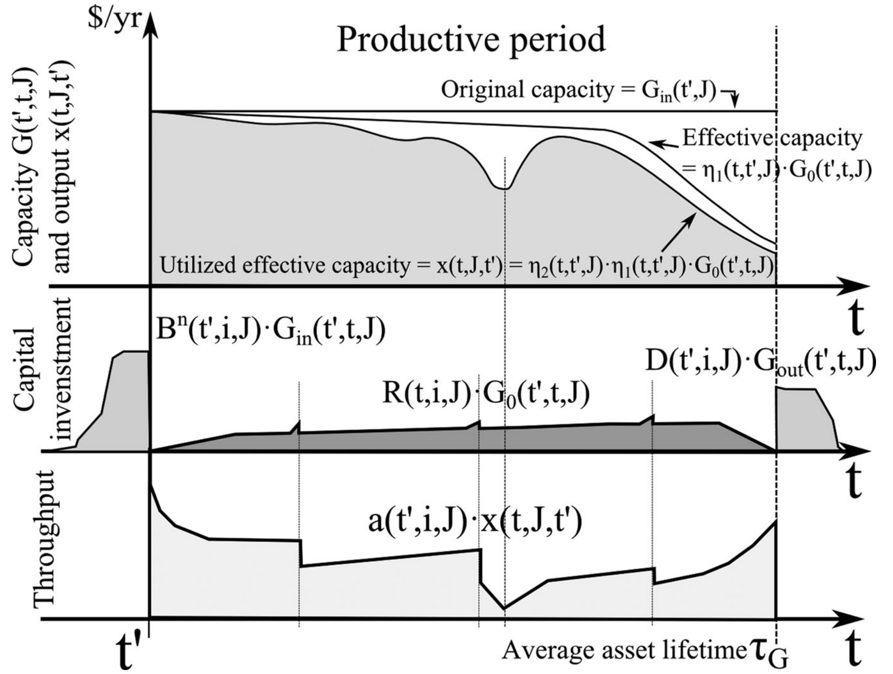

Figure 2 Overview of the parameters that describe the life cycle of the factories in sector J that were built in year t' and that are demolished in year t'+ τ_G.

different capital goods in an asset have different lifetimes and their presence may affect the age efficiency of the asset, there exists a relation between capacity G, capital content C, age efficiency η_1, capital lifetime λ_C, and the maintenance matrix R. This relation can have any degree of complexity, and it is the task of the model developer to maintain a level of detail that is necessary and sufficient to tackle the research question at hand.

A Dynamic Input-Output Model with Age Cohorts of Assets

A directed, bipartite graph is the common system structure behind the supply-and-use framework (UN 2008), I/O models (Miller and Blair 2009), integrated assessment models (Loulou et al. 2005), and general equilibrium models (Burfisher 2011) (figure 3). All processes in the system can be divided into two groups: industries and markets. All flows in the system begin at an industry and end at a market or vice versa (bipartite property); they represent commodity flows. Environmental extensions, including natural resources or emissions, are not considered here. We assume that there is a one-to-one correspondence between products and industries, so that the supply table is square and diagonal. That means that each industry J produces exactly one commodity j, and in the equations below, we use the index J to denote both industries and commodities. To resolve the issue of where to draw the boundary between stocks and flows or throughput and capital the concept of the asset boundary was introduced. It includes a definition of the time interval over which the industrial metabolism is discretized, which is called the accounting period, and which is typically *1 year* (UN 2008). The accounting period divides the interindustrial flows into throughput (gray flows in figure 3) and investment (black flows).

The markets in the system in figure 3 obey a general balance equation, which can be read directly from the system definition (equation (10)).

$$x = Ax + \underbrace{B^n G_{in} + RG_0 + DG_{out}}_{GFCF-dwellings} + \tilde{y} \qquad (10)$$

Here, $\tilde{y}$ denotes the net final demand that does not contain investment into construction, maintenance, and demolition of industrial assets. The market balance in equation (10) is the starting point for the Leontief primary model. It can be solved

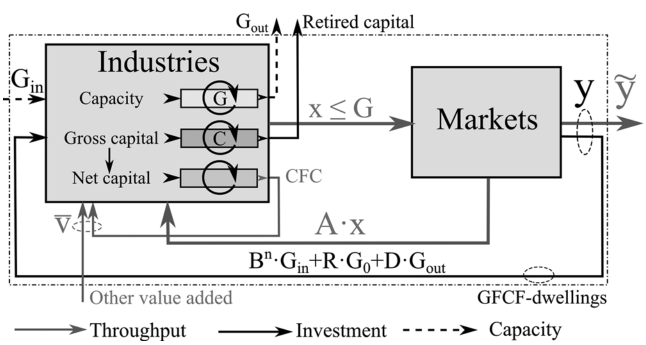

Figure 3 Definition of the input-output system. The system drawing contains the most central system variables listed in table 1. Capacity, gross capital, and net capital are three aspects of industrial assets or fixed capital. They are accounted for in parallel.

for the total industrial output x, provided that all other variables are known. We now use the capacity modeling framework to formulate a dynamic I/O model that is driven by final demand. We assume that for each year of the modeling period, an exogenous net final demand $\tilde{y}$is given. We assume the capital intensity matrices B^n, R, and D to be known from process inventories of current and future technologies, and we assume lifetimes $\lambda_G(t, t', J)$, age efficiencies $\eta_1(t, t', J)$, load factors $\eta_2(t, t', J)$, the age-cohort–specific technical coefficients $a(t',i,J)$, and the age structure of the original productive capacity in a starting year, $G_0(t = t_0, t', J)$, to be given:

Given: $\tilde{y}, R, B^n, D, \lambda_G, \eta_1, \eta_2, a, G_0(t = t_0)$, to be obtained: x, A, G_{in}, G_{out}

New capacity $G_{in}(t = t', J)$ and retiring capacity $G_{out}(t, t', J)$ in industry J are connected by a dynamic capacity model as introduced above (equation (11)):

$$\begin{aligned} G_{out}(t, t', J) &= G_{in}(t', J) \cdot \lambda_G(t, t', J) \\ G_0(t, t', J) &= G_{in}(t', J) \cdot \left(1 - \sum_{t' \leq t'' \leq t} \lambda_G(t'', t', J)\right) \end{aligned} \tag{11}$$

We can now synthesize the interindustry flow matrix Z by scaling the technical coefficients of the individual assets of different age cohorts $a(t',i,J)$ by their respective output $x(t,J,t')$ and by relating output to capacity using equation (7) (equation (12)):

$$\begin{aligned} Z_{iJ}(t) &= \sum_{t'} a(t', i, J) \cdot x(t, t', J) \\ &= \sum_{t'} a(t', i, J) \cdot \eta(t, t', J) \cdot G_0(t, t', J) \end{aligned} \tag{12}$$

where $\eta(t, t\prime, J)$ is defined as the product of capacity utilization and age-efficiency (equation (13)):

$$\eta(t, t', J) = \eta_2(t, t', J) \cdot \eta_1(t, t', J) \tag{13}$$

The industry-wide attributional A-matrix A^a can be synthesized from the technical coefficients of the individual age cohorts of assets and their respective utilization rates and age efficiencies (combination of equations (12) and (7)), as shown by equation (14):

$$A^a_{iJ}(t) = \frac{Z_{iJ}(t)}{x_J(t)} = \frac{\sum_{t'} a(t', i, J) \cdot \eta(t, t', J) \cdot G_0(t, t', J)}{\sum_{t'} \eta(t, t', J) \cdot G_0(t, t', J)} \tag{14}$$

Equation (14) combines process inventories with dynamic stock modeling and represents a synthesis of the average A-matrix of technical coefficients from process inventories of assets of different age cohorts. The possibility of such a synthesis was pointed out by several researchers (Carter 1963; Idenburg and Wilting 2000; Lennox et al. 2005; Pan 2006; Ryaboshlyk 2006).

We now describe the model solution. The following calculations are to be performed stepwise, year by year, starting in the first model year $t_0 + 1$. The scheme of calculations is shown in figure 4.

First, we determine the retiring capacity $G_{out}(t, t', J)$ according to equation (11) and subtract it from the existing capacity stock at the end of the previous year *t-1* and denote the thusly obtained intermediate capacity by $\tilde{G}_0(t, t', J)$. At the end of each model year, the new capacity is added to the stock (equation (15)).

$$\begin{aligned} \tilde{G}_0(t, t', J) &= G_0(t-1, t', J) - G_{out}(t, t', J) \\ G_0(t, t', J) &= \tilde{G}_0(t, t', J) + G_{in}(t' = t, J) \end{aligned} \tag{15}$$

By substituting $x(t, J)$ with equations (7) and (15), we can reformulate the market balance equation (10) at the end of each model year as shown in equation (16).

$$\begin{aligned} &\sum_{t'} \left(\eta(t, t', J) \cdot \left(\tilde{G}_0(t, t', J) + G_{in}(t' = t, J)\right)\right) \\ &= \tilde{y}(t, J) + \sum_{J'} \begin{pmatrix} A(t, J, J') \cdot \sum_{t'} \left(\eta(t, t', J') \cdot \left(\tilde{G}_0(t, t', J') + G_{in}(t' = t, J')\right)\right) + B^n(t, J, J') \cdot G_{in}(t, J') \\ +R(t, J, J') \cdot \sum_{t'} \left(\tilde{G}_0(t, t', J') + G_{in}(t' = t, J')\right) + \sum_{t'} D(t', J, J') \cdot G_{out}(t, t', J') \end{pmatrix} \end{aligned} \tag{16}$$

We solve equation (16) for $G_{in}(t, J)$ and combine the thusly obtained equation with equation (14) for the A-matrix and equation (15) for the capacity balance. This leads to an equation system that has to be solved for each model year in turn. It is shown in section S1-2 in the supporting information on the Web.

If a unique solution for $G_{in}(t, J)$ and $A(t)$ can be obtained using an iterative approach similar to the one designed by Lennox and colleagues (2005), $x(t)$ can be determined by solving the market balance in equation (10) for the total required industry output (equation (17)):

$$\begin{aligned} x(t) = (I - A(t))^{-1} \cdot (B^n(t) \cdot G_{in}(t) \\ + R \cdot G_0 + D \cdot G_{out} + \tilde{y}(t)) \end{aligned} \tag{17}$$

Performing these calculations for all model years from t_0+1 to the time horizon of the model yields a time series of A-matrices and industry output x.

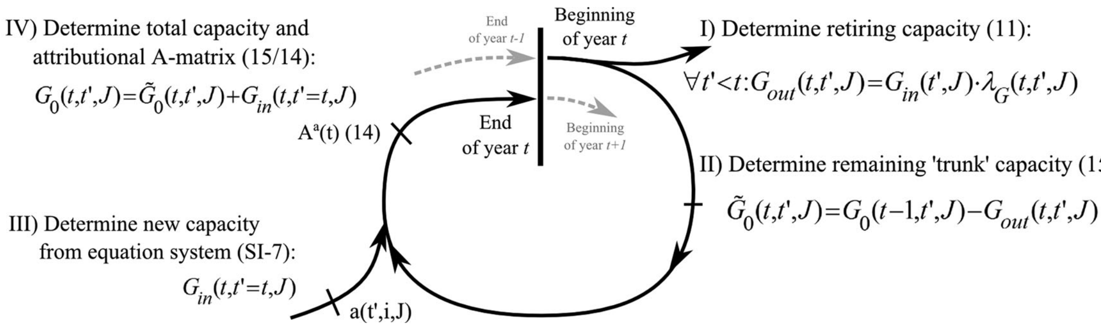

Figure 4 Scheme of calculations for the year-by-year loop. Equation numbers are shown in brackets.

Future Application: Economic and Physical Models of the Industrial Metabolism

Combined economic and physical models of industrial flows have a long tradition, as stated in the *Introduction*. Combining the physical and economic layers of dynamic capital stock models allows us to consider several important issues: First, material requirements of new fixed assets may alter global material cycles because they may be much more material intensive than established technologies (Vidal et al. 2013). Increasing metal demand from the installation of new energy technologies on the large scale may require mining of lower-grade ores, which may be substantially more energy intensive than present mining activities (Norgate and Jahanshahi 2006; Northey et al. 2014). Second, new energy technologies often rely on specialty materials, which may turn into bottlenecks when new energy technologies are deployed on the large scale, because the availability or the access to some mineral resources may be limited (Graedel et al. 2012). Finally, recycling scrap from retiring fixed assets could reduce primary material production and related impacts associated with primary production. Dynamic stock modeling allows for estimating the future generation of waste for recycling and treatment that arises from retiring fixed assets. Monetary data for capital stocks are more readily available than information on their material content and could thus help to estimate the material stocks in fixed capital assets as well as material inflows and outflows using equation (6).

The material layer of industrial and other in-use stocks and the material layer of interindustry flows, resources, and emissions together form a full representation of the anthropogenic material cycles. Material flows between industries and waste treatment activities can be described with the WIO model and the WIO-MFA approach (Nakajima and Nakamura 2006). WIO-MFA is also well suited to determine the array of material concentration μ of capital goods (Nakamura et al. 2007). Combining a WIO model that was amended with resource and emissions accounts with a dynamic capital stock model would allow us to perform a detailed, economy-wide dynamic MFA, where the WIO model describes interindustry flows and the dynamic stock model describes the fixed capital stocks and their development over time.

Recent developments in economic accounting and I/O modeling allow for establishing a mass balance not only for product markets, but also for industries (Schmidt et al. 2010; Nakamura et al. 2007; Giljum and Hubacek 2009). When constructing an IO model from a supply-and-use table, one has to consider that the system-wide production balance and the industry balance may be distorted by different constructs (Majeau-Bettez et al. 2014), and the allocation of resource use, waste, and emissions, to single-output processes may break the mass balance of industries (Weidema and Schmidt 2010).

What Information is Required?

National accounting, I/O, and MFA have different foci, and additional effort or assumptions in bridging the data requirements between these frameworks may be required. Because we operate at the economy-wide level, some generic representation needs to be given for all industries, irrespective of the level of detail of certain materials and their relevant sectors. We can expect that top-down macro-level estimates will have to be refined into meso-level or finer assessments, for example, for a particular metal type (Hawkins et al. 2007).

We break down the data requirements into production recipes, capacity information, and output/demand information. Production recipes in the form of historic coefficient matrices $a(t',i,J)$ are readily available from I/O databases, and future or best available technology inventories are available for integration from LCI databases (Gibon et al. 2014) (stocks as consumption couplers). Matrices of capital formation to estimate K or B are also readily available from national accounting institutes or through projects such as KLEMS (O'Mahony and Timmer 2009) (stocks as capital containers). The split of K into K_B, K_R, and K_D, or B^n, R, and D, is generally less readily available. As in previous work on this topic, the split can be made by industry knowledge coupled with expert judgment. Examples from the literature are given in table S1-1 in the supporting information on the Web. Capacity information (lifetime, age efficiency, and capacity utilization) can be gleaned from national statistics, although we acknowledge the lack of transparent, easily accessible data here. Expert judgment on proxy data can often be utilized. Finally, drivers of industrial activity $\tilde{y}$ are readily available in national statistics and I/O databases for historic years and can be linked to exogenous scenario projections common in integrated assessment models or other forward-looking models. G_0 is readily available from national statistics. Data on material content or unit price μ of products can be obtained from trade statistics, product datasheets, and product composition databases. The material content of productive assets can sometimes be obtained from company statistics (stocks as resource repositories). As a field, IE is well positioned to supplement economic statistics with physical aspects of the industrial metabolism, as it has contributed to the provision of publicly accessible work in the field of environmentally extended multiregional IOA.

Other Applications

We discuss how the framework and the IO model can be applied to consequential modeling and how it relates to model families with elaborated treatment of markets, such as integrated assessment and general equilibrium models.

Capital Stocks and Consequential Life Cycle Assessment

CLCA aims at quantifying the consequences of a decision within an economy (Zamagni et al. 2012). Here, we consider an increase in final demand, which represents a perturbation of the industrial system that affects both markets and industries. According to Schmidt (2008, 352), the consequential approach "implies that marginal, i.e. actual affected processes are included [. . .]." Unlike in neoclassical economics, markets in the Leontief IO framework do not play an active role; they merely match industry supply to intermediate and final demand. Both product transformation and price formation are attributed to the industries. The dynamic model of production capacity can be used to model the consequences of an increase in final demand for industrial output and capacity extension in a Leontief IO framework. Figure 5a shows the response of the industrial system to a change in final demand. The markets in the system were balanced in their original state (black variables), and hence the marginal changes alone balance as well (gray variables). An increase in industry output x_C can be met by (1) increasing the utilization rate of existing capacities or (2) by installing new capacity (figure 5b). In the first case, one needs to know the split between fixed costs a_1 and variable costs a_2 of an asset. Fixed costs are related to the capacity, irrespective of its utilization rate, and variable costs are coupled to the actual output. The increase of the utilization rate of existing assets $\Delta\eta_2$ determines which part of the additional demand can be delivered by existing capacities. In the second alternative, the capital intensity B^n, the fixed and variable costs per unit of output from the new assets $a_1(t\prime = t, i, J)$ and $a_2(t\prime = t, i, J)$, and the utilization rate η_{2_C} of the new assets need to be known. One can then establish and solve an equation system that determines the change in output x_C as a function of the change in the utilization rate of the existing capacities and the utilization rate of the marginal assets as well as their capital intensity and fixed and variable costs. The equations are shown in section S1-6 in the supporting information on the Web.

The dynamic capacity model allows us to allocate the change in output x_C to specific age cohorts and technologies of productive capacity by the choice of $\Delta\eta_2$ and η_{2_C}. The environmental consequences of producing x_C can thus be associated with specific assets, and not with average technology, as in the attributional approach. To endogenously determine utilization rates and their changes resulting from increasing demand, the Leontief IO framework needs to be extended, because more sophisticated modeling of industries and especially markets is required. Examples include the use of partial or general equilibrium models (see Earles and Halog [2011] for an overview and Whitefoot et al. [2011] for an example), a detailed study of the likely market response depending on the magnitude of change in industrial output (Weidema et al. 2009; Schmidt 2008), or agent-based modeling (Axtell and Andrews 2002). The framework presented here does not represent an alternative to previous concepts; it should rather be seen as a versatile descriptive model of the fixed capital stock that can couple the I/O framework to different market-driven consequential approaches.

The Connection to Technology-Rich Integrated Assessment Models and General Equilibrium Models

Some integrated assessment models (IAMs) contain detailed descriptions of capital stocks including age-cohorts and different technology types, which are very similar to the concepts described here. The TIMES (The Integrated MARKAL-EFOM System) model is a good example (Loulou et al. 2005). Because these models often minimize total costs, the technical coefficients of fixed assets are not necessarily constant, but can vary over a certain range. This provides more flexibility in the modeling of product substitution, but poses challenges regarding the physical balances of the industrial processes in these models, because the material layer of the industrial metabolism is not consistently covered. Computable general equilibrium models consider perfectly competitive product and labor markets. They use nested production functions, which assume some substitutability between different production factors, but limited substitutability between intermediate requirements (Burfisher 2011). This model class is timeless because it only computes equilibrium states. The capital stock is modeled on an abstract and aggregate level only; no age cohorts or materials are tracked.

For scenario modeling of long-term resource use, emissions, and waste flows, a physically balanced model is indispensable. No matter how products and production factors are distributed between end users and producers (economic layer), physical balances should always be respected by the models, because they represent an insurmountable constraint to a transition to a low-carbon society. Especially when modeling on the large scale, physical constraints represent challenges when, for example, the extent of material recycling is constrained by the available amount of recyclable material. The simultaneous

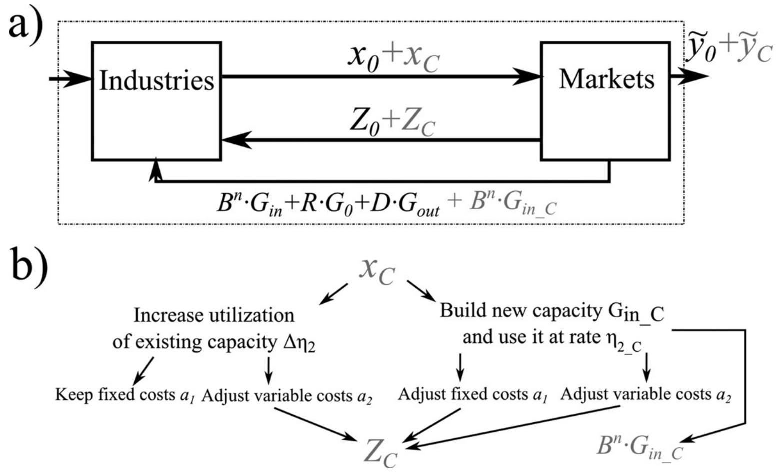

Figure 5 (a) Original state (black) and perturbations (gray) of an industrial system after an increase in final demand. (b) The split of the additional output x_C into existing and new capacities and the resulting interindustrial flows Z_C.

age-cohort-lifetime–based accounting of capital and material stocks can be combined with assessment methods other than dynamic I/O analysis. This would allow for more sophisticated modeling of market mechanisms than what is possible in I/O. It could lead to the development of comprehensive and physically balanced models of industries and markets, which can be used to build scenarios for the future socioeconomic metabolism under different economic paradigms.

Conclusion

Under global resource and emissions constraints, deliberate management of stocks over long time intervals may be a key strategy to maintain high levels of human well-being (Boulding 1966). In such a world, material stocks should be tracked as carefully as capital stocks to facilitate the estimation of future mineral resource use related to industrial assets and the potential for material recovery and recycling from obsolete capital stocks (stocks as capital containers and resource repositories).

The turnover speed of the existing assets determines how quickly new technologies can replace old ones (Davis et al. 2010) (stocks as dynamics determiners). Tracking age cohorts is a generic and versatile way for modelers to reflect the inertia that capital stocks represent. Age-cohort–based accounting of stocks has a long tradition in capital stock measurement and MFA. A coordinated effort may enable researchers and accountants to synchronize and harmonize their accounting frameworks to provide a more detailed and reliable understanding of the dynamics of industrial efficiency (stocks as consumption couplers) and the capital and material requirements needed to build a sustainable energy future (IEA 2010).

The modeling and accounting framework presented here can serve as a guideline for future modeling efforts. It brings together different aspects of socioeconomic metabolism related to in-use stocks, which, at present, are covered by the different disciplines, MFA, national accounting, IOA, and LCA. We showed that there is substantial overlap between those fields and identified synergies that may be exploited in future work.

Acknowledgments

The research presented here was made possible by the European Commission under the DESIRE Project (grant no.: 308552). Any opinions presented and conclusions drawn reflect the authors' point of view and not necessarily that of the European Commission. The authors thank Guillaume Majeau-Bettez and three anonymous reviewers for their valuable suggestions.

Notes

1. Note that the elements of D can be negative, which represents the salvage value of the retiring assets.
2. Miller and Blair (2009) denote the capital stock by K, whereas, here, we use C for the stock of gross fixed capital and K for the flows into fixed capital.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's web site:

Supporting Information S1: This supporting information provides a detailed literature study, some calculation details, an extended model with subsector technology specifications, and applications for consequential LCA.