Economic Systems Research

ISSN: 0953-5314 (Print) 1469-5758 (Online) Journal homepage: http://www.tandfonline.com/loi/cesr20

UNCERTAINTY ANALYSIS FOR MULTI-REGION INPUT–OUTPUT MODELS – A CASE STUDY OF THE UK'S CARBON FOOTPRINT

Manfred Lenzen , Richard Wood & Thomas Wiedmann

Economic Systems Research, 2010, Vol. 22(1), March, pp. 43–63

UNCERTAINTY ANALYSIS FOR MULTI-REGION INPUT–OUTPUT MODELS – A CASE STUDY OF THE UK'S CARBON FOOTPRINT

MANFRED LENZEN[a*], RICHARD WOOD[a,c] and THOMAS WIEDMANN[b]

[a]*Centre for Integrated Sustainability Analysis, A28, The University of Sydney, NSW 2006, Australia;* [b]*Stockholm Environment Institute, University of York, UK;* [c]*Industrial Ecology Program (IndEcol), NTNU, Trondheim, Norway*

(*Received 17 February 2009; In final form 5 January 2010*)

This paper reviews and demonstrates methods available for estimating standard deviations for carbon multipliers in a multi-regional input–output (MRIO) framework. We attempt to capture all possible variations of underlying data and calculation procedures in a global MRIO model constructed with particular focus on the UK. We consider these variations to be random, and determine the stochastic variation of the whole MRIO system using Monte Carlo techniques. 5000 simulation runs were carried out to determine the standard deviations of multipliers. From these, the standard deviations of components of the UK's carbon footprint were estimated using error propagation. We estimate an 89% probability that the UK's carbon footprint has increased between 1994 and 2004.

Keywords: Uncertainty; Multi-region input–output analysis; Monte Carlo analysis; Errors; UK; Carbon footprint

1. INTRODUCTION

In a parallel paper in this journal, Wiedmann et al. (2010) describe the construction of, and results from, a multi-regional input–output model of the United Kingdom (UK-MRIO). The aim of their work for the UK's Department of Environment, Food and Rural Affairs (Defra) was to set the basis for multi-country analyses of environmental impacts associated with UK trade flows, including detailed accounts of emissions embodied in trade flows to and from the UK. A time series of direct and indirect carbon dioxide emissions associated with UK economic activities was calculated, with particular emphasis on emissions embodied in imports to and exports from the UK.

The results documented in Wiedmann et al. (2010) indicate that – contrary to prior belief – the UK's climate change responsibility had increased over the past decade, because emissions-intensive production was outsourced to other countries. Given the political implications of communicating these initial results to the public, the authors were asked to supply Defra with an estimation of the uncertainty associated with their findings. This estimation is the subject of this paper.

In Subsection 1.1, we review background, and methods available for estimating the standard deviations of multipliers derived from an MRIO framework. In Section 2 we

*Corresponding author. E-mail: m.lenzen@physics.usyd.edu.au

ISSN 0953-5314 print; ISSN 1469-5758 online
DOI: 10.1080/09535311003661226

then describe in detail the methodology applied to the work for the UK Government. This methodology consists of estimating standard deviations for the raw data, for the extended MRIO table entries, for the carbon multipliers, and for the components of the final carbon footprint. Section 3 presents the results of the work for Defra on the UK's carbon footprint. Section 4 discusses the validity of the assumptions and calculations, and concludes. For the definitions, general context and a detailed description of the UK-MRIO model and results we refer to the main report by Wiedmann et al. (2008).

1.1. Literature Review

Soon after Leontief's initial publications, researchers started to apply uncertainty calculus to input–output analysis. One prominent technique that has been utilised is Monte Carlo analysis (Bullard and Sebald, 1988).[1] This is because researchers recognised that Leontief's basic input–output relationship $\mathbf{x} = (\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{y}$, linking final demand $\mathbf{y}$ with gross output $\mathbf{x}$, cannot be differentiated analytically with regard to multiple elements a_{ij} of $\mathbf{A}$ (Quandt, 1958, 1959; Bullard and Sebald, 1977).

In the majority of Monte Carlo analyses of input–output systems, researchers have assumed that the uncertainty of basic input–output data can be formulated in terms of normally distributed, uncorrelated, stochastic errors, with defined standard deviations (Quandt, 1958; Goicoechea and Hansen, 1978; Hanseman and Gustafson, 1981; Hanseman, 1982; Lenzen, 2001). In a nutshell, these standard deviations are sourced for components of $\mathbf{y}$ and of $\mathbf{A}$, and used for perturbing $\mathbf{y}$ and $\mathbf{A}$ to $\mathbf{y}^*$ and $\mathbf{A}^*$. From the latter, perturbed input–output multipliers $\mathbf{m}^* = (\mathbf{I}-\mathbf{A}^*)^{-1}$ are calculated, and compared with the initial multipliers $\mathbf{m} = (\mathbf{I}-\mathbf{A})^{-1}$ (Sakai *et al.*, 2000). Relative standard deviations $\Delta\mathbf{m} = (\mathbf{m}^*-\mathbf{m})/\mathbf{m}$ for these multipliers are then estimated from typically many thousand perturbation runs (Evans, 1954; Park, 1973; Bullard and Sebald, 1977). Monte Carlo techniques have also been applied in Life-Cycle Assessment (LCA), where researchers use generalised input–output systems that link a monetary table with physical satellite accounts (Hondo and Sakai, 2001; Lenzen, 2001; Nansai et al., 2001; Yoshida et al., 2001; Hondo et al., 2002; Yoshida et al., 2002; Peters, 2007a).

Some authors object to the assumption of uncorrelated, normally distributed error terms (Stevens and Trainer, 1980; Park et al., 1981; Rey et al., 2004), for example because the data collection and balancing procedures followed by national statistical agencies are affected by systematic underreporting, or because the accuracy of row and column totals implies that if one element is too large, others in the same row or column must be too small. Others examine effects of error data that are distributed other than normally, and correlation structures between the errors of certain entries a_{ij} in $\mathbf{A}$, on resulting standard deviations $\mathbf{dm}$ for input–output multipliers (West, 1986; Jackson and West, 1989; Kop Jansen, 1994; Ten Raa and Steel, 1994). Whilst these possibilities and effects cannot be ruled out, there is at present no data that would allow their incorporation into real-world case studies.

In general practice, only a minor proportion of authors actually add uncertainty analyses to their input–output case studies. In particular, there is a dearth of environmental MRIO

[1] As an alternative, fuzzy set theory has also been employed to estimate the uncertainty of an IO system used to calculate Ecological Footprints (Beynon and Munday, 2008).

studies where an uncertainty analysis is undertaken that is similarly thorough as the one presented in this paper (Wiedmann et al., 2007; Wiedmann, 2009). Lenzen et al. (2004) examine the effect of two types of errors on Danish carbon multipliers and trade balances: the effect of the omission of feedback facilitated by international trade, and sector aggregation. Whilst the inclusion of Danish exports led only to minor corrections, the explicit modeling of Danish imports, as well as sector disaggregation were concluded to be important for overall accuracy.

Weber and Matthews (2007) vary their MRIO calculations by using different input parameters for two of the major uncertainties in their model – the Rest-of-World approximation and the choice between market exchange rates and Purchasing Power Parities – and present 'feasible ranges for EEE and EEI' (Weber and Matthews, 2007, p. 4876), but they do not carry out a Monte Carlo analysis. Weber (2008) presents a detailed discussion and empirical investigation of uncertainties in MRIO modeling. Weber examines three major uncertainties by using a series of models built using input–output data from the United States and seven of its largest trading partners. They relate to aggregation and concordance to a common sector scheme, treatment of the rest-of-world (ROW) region, and monetary exchange rates. These are MRIO-specific sources of error that add to uncertainties in standard, single-region input–output analysis (Weber, 2008, p. 27, Table 3). Weber (2008) states that '... it is likely that these inherent uncertainties often end up raising total uncertainty beyond the levels of a detailed (i.e. >200 sector) single-region model' and concludes that '...detailed single region models with simplified trade modeling should also be considered, especially if the analysis only requires a few commodities to be modeled and a hybrid analysis using SPA[2] is possible.'

Aggregation is a problem in particular when high and low impacting sectors are combined in one aggregated sector. Examples are pulp/paper and publishing, cement and non-metallic minerals, post and telecommunications (for these three see Weber, 2008), or aluminium and other non-ferrous metals. Lenzen et al. (2004) analyse the error associated with aggregation by merging their 39–133 sector MRIO to 10 aggregated sectors and find significant errors, particularly due to aggregating electricity together with gas and water production. The sensitivity of MRIO calculations towards the use of MER or PPP as a means to convert currencies is tested by Weber and Matthews (2007). Weber (2008) finds that the ratio of the rates (MER/PPP) can be as high as 4.7 for China versus the US and contemplates the use of hybrid currencies within the compound **A** matrix, along with sector-specific exchange rates (see also Lenzen et al., 2004). The trade flow matrices, i.e. the off-diagonal elements in a full, multidirectional MRIO model, also deserve special attention. Imports from one region to another are, in most cases, not known as matrices to using sectors, but only as vectors from supplying sectors. Imputation techniques with inherent assumptions are required to produce these trade flow matrices. One solution to the estimation problem for off-diagonal trade flow matrices is to use trade coefficients (Lenzen et al., 2004). This procedure assumes that the trade coefficients are identical for all entries along a row of the imports matrix, i.e. for all using domestic industries – clearly an assumption with considerable impact on the accuracy of MRIO models.

---

[2] Structural Path Analysis (see e.g. Defourny and Thorbecke, 1984; Treloar, 1997; Lenzen, 2006; Peters and Hertwich, 2006).

2. METHODOLOGY

In the following we establish uncertainty estimates for CO_2 multipliers **m** calculated from an MRIO table **T** according to $\mathbf{m} = \mathbf{q}[\mathbf{I} - \mathbf{T}\hat{\mathbf{x}}^{-1}]^{-1}$, where **q** are sectoral CO_2 intensities, **x** is sectoral gross output, and **I** is a suitable identity matrix. These uncertainties are expressed as standard deviations (SDs) **dm**, denoting the 67% confidence interval around a mean value of **m**. In other words, 67% of a large number of observations of **m** would fall into the interval [**m**−**dm**, **m**+**dm**] (and furthermore, 95% of observations are captured by $\pm$2dm and 99% by $\pm$3dm). In order to calculate the **dm**, it is necessary to determine the SDs **dT** of the MRIO table **T** itself, which stem directly from uncertainties in the raw data. Hence, the calculations carried out in this work proceed in three major stages: (1) determine SDs of raw data; (2) determine SDs **dT** of **T**; and (3) determine SDs **dm** of **m**. Before proceeding with the description of these stages, we make three general remarks.

First, we define the *order of magnitude* of a data item x as its logarithm $\log_{10}(x)$. For example, the order of magnitude of £1000 is 3, the order of magnitude of £10,000 is 4. Let the absolute SD of a data item x be dx, so that its Relative Standard Deviation (RSD) is $r_{\mathrm{x}} = \mathrm{d}x/x$. The absolute error in the logarithms (the 'order-of-magnitude' error) can be approximated by

$$\mathrm{d}(\log_{10} x) \approx \log_{10}(x + \mathrm{d}x) - \log_{10}(x) = \log_{10}\left(\frac{x + \mathrm{d}x}{x}\right) = \log_{10}(1 + r_x). \tag{1}$$

RSDs r_x can sometimes be obtained or derived from public data sources. If there are no data, RSD estimates can be solicited from informed judgement of statistical agency staff (Lenzen, 2001). The rationale for estimating SD coefficients for the logarithms and not for the data values as such is based on the assumption that observations of MRIO entries are distributed log-normally, and not normally. The assumption of log-normality effectively ensures that Monte Carlo-perturbed entries never change sign. Had we used normal distributions instead, their lower tails would have extended towards negative values, which pose problems for input–output analysis (Ten Raa and Van der Ploeg, 1989; Kop Jansen and Ten Raa, 1990). In that case, the only solution would have been to truncate these negative distribution tails; this however would have led to undesirable bias in the Monte Carlo perturbations, and to artificially skewed distributions for the multipliers.

Second, we deal with the fact that in many cases, SDs are not available for the raw data used to construct the UK-MRIO table, by using SDs available for proxy data. More specifically, we use the pattern of these proxies and their SDs to infer the SDs of the raw data. In general, raw and proxy data and their SDs exhibit the same following trends: large data items are generally known with higher accuracy than small data items. This is due to the fact that large items often consist of many smaller data points (for example purchases by companies, emissions data from individual sources, etc). Provided there is no correlation between table elements, or in other words no systematic error component, stochastic errors cancel out during aggregation. This holds in general for any type of data: for example, employment figures in the retail and service sectors of an economy are large, because the totals consist of many businesses each employing a small number of people. Another example is energy use in the electricity sector. Even though there may not be many power plants in the country, the amount of black coal or natural gas is usually comprised of many supply batches adding up over the year. The inference of the raw data SDs from the proxy data

SDs can be facilitated using regression techniques. In this work we tried several specifications for regressing the relative standard deviation of the raw data and found that a power regression with two coefficients, a and b, yielded the best fit in all cases:

$$r_{\mathrm{x}} = a\mathrm{x}^{b} \tag{2a}$$

We then calculate the log-normal standard deviation as well as the regressed order-of-magnitude error against the raw data x according to

$$log_{10}(1 + r_{\mathrm{x}}) = log_{10}(1 + a\mathrm{x}^{b}) \tag{2b}$$

Third, although our uncertainty analysis of the UK-MRIO model tries to capture all possible stochastic variations of underlying data and calculation procedures, we do not deal with possible systematic error sources such as structural changes and sectoral price changes in foreign IO data over time, systematic over- and underestimation of CO_2 intensities of foreign industries due to the mismatch of sectors in UK and foreign IO and CO_2 data, change of import structure over time, or choice of currency conversion factors.

2.1. Uncertainties of Raw Data

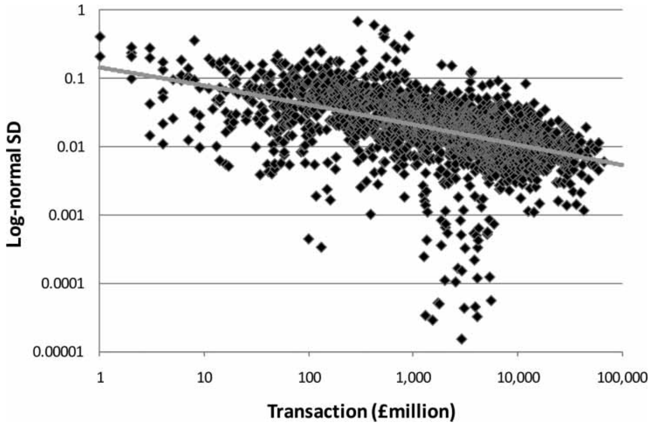
In order to perform a Monte Carlo analysis of the UK-MRIO model, the uncertainties of all underlying input data had first to be determined. There are six types of data with their specific uncertainties:

- UK input–output data;
- UK CO_2 emissions data;
- input–output data for three world regions;
- Producer Price Indices (PPI) to be used as deflators to accommodate structural change;
- CO_2 emissions data for three world regions; and
- international trade data.

First, input–output data for the UK were taken from the annual Supply and Use Tables (SUT) reported by the Office for National Statistics (ONS, 2007c) and supplemented for the purpose of the UK-MRIO as described in (Wiedmann et al., 2008). No explicit publication of the uncertainty associated with UK IO data is available, but the Annual Business Inquiry (ABI) published by ONS (ONS, 2007b) includes an estimate of the standard error (SE) associated with its underlying data used to compile IO accounts. More precisely, the SE is reported for 'Total turnover', 'Approximate gross value added at basic prices', 'Total purchases of goods, materials and services', and 'Total net capital expenditure' at a SIC 3-digit level for the years 1998 to 2005. We regard the 2166 data points x and absolute standard errors Δx from this publication as representative for the error associated with UK input–output data in general and used them as the basis for regressing the SDs for all SUT entries. These data provide an excellent example for decreasing SDs with increasing data values (Figure 1).

Second, UK CO_2 emissions data were taken from the UK Environmental Accounts (ONS, 2007a) which, in turn, are based on the official UK Greenhouse Gas Inventory submitted under the Framework Convention on Climate Change (Jackson et al., 2009b).

FIGURE 1. Standard deviations for UK input–output data regressed from Annual Business Inquiry data (ONS, 2007b).

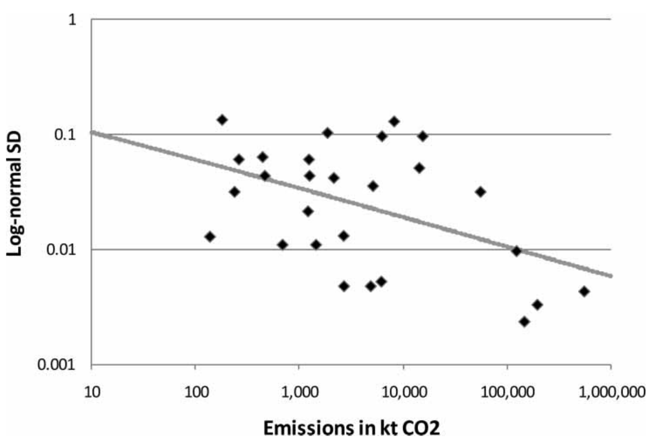


Note: $r_x = 0.393x^{-0.302}$, $R^2 = 0.257$.

The latter provides an estimate of uncertainty at a broad sectoral level, using an error propagation approach (Jackson et al., 2009a, pp. 491–498). The resulting standard deviations of this approach have also been regressed according to equation (2) (Figure 2, 27 data points).

Third, we used the input–output database of the Global Trade Analysis Project (GTAP 6; Dimaranan, 2006) for three world regions in our UK-MRIO. GTAP performs a number of process steps that influence the accuracy and precision of their data. Peters (2008a) describes the procedure: input–output data are contributed to GTAP voluntarily and can be rather outdated. GTAP scales the data to match GDP in international dollars, which means the data have the structure of its base-year, but the volume of the target year. The conversion

FIGURE 2. Standard deviations for UK CO_2 emissions estimates (adapted from Table A 7.6.1 in Jackson et al. 2009a, p. 495).



Note: $r_x = 0.486x^{-0.261}$, $R^2 = 0.212$.

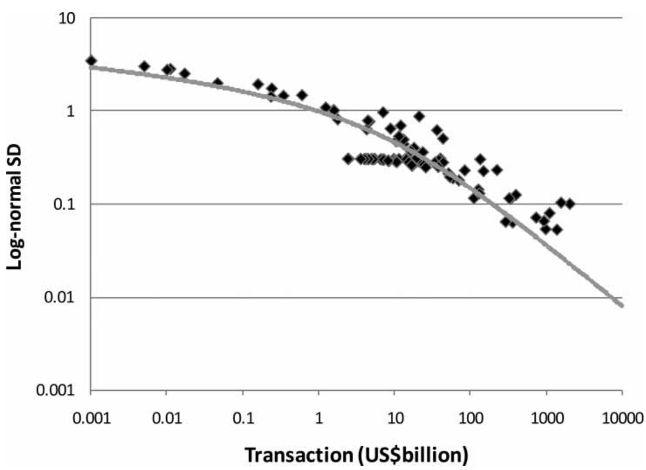
of original data into a harmonised GTAP format requires various aggregations and disaggregations. The error associated with these procedures is unknown but potentially significant, in particular when the original data are significantly more aggregated. GTAP also includes additional data (trade and energy), removes other (changes in stocks), and then re-balances the entire global account. Despite these obviously significant error sources, GTAP data are widely accepted as a reputable source for economic analyses. However, in most economic studies no uncertainty analysis is provided.

To estimate SDs for the UK-MRIO we used GTAP information on the extent of change in the structure of individual tables that were 'fitted' in order to accommodate international data such as trade targets, energy usage targets, and other international data sets (McDougall, 2006, Table 19.6: 'Selected category total comparisons between unfitted and fitted input–output tables'; more than 100 data points at the sector level). Assuming that the differences between 'unfitted' and 'fitted' represent a SD proxy, we achieve a reasonable regression (Figure 3).

Although the comparison between 'fitted' and 'unfitted' values by GTAP is confined to row and column totals, not individual cells, the authors also state that they 'select those [pairs of values] that make the largest contribution to the entropy distance measure [...]. This identifies large sectors in large regions with large relative changes.' (McDougall, 2006, p. 19–9) We therefore regard our regression in Figure 3 as an upper (conservative) estimate of uncertainties in foreign IO data. Peters (2007b) discusses in detail the uncertainty associated with the GTAP 6 database (see also appendices in Reinvang and Peters, 2008, and WWF Trade and Investment Programme and Norwegian University of Science and Technology Industrial Ecology Programme, 2008).

Fourth, the UK-MRIO uses GTAP data from 1997 and 2001 for the creation of a time series from 1992 to 2004. In reality however, the economic structure – and thus the input–output tables – will have changed over time. Although it can be assumed that these changes are of a systematic nature (that is, changes are likely to go in one direction, for

FIGURE 3. Distribution of differences (expressed as standard deviation) between original ('unfitted') IO tables and those 'fitted' by GTAP (McDougall, 2006).



Note: $r_x = 8.927x^{-0.672}$, $R^2 = 0.863$.

example sectors become consistently either more or less important over time), we do not make sector-specific assumptions about the directions of change. Rather, we consider the changes observed between 1997 and 2001 as stochastic uncertainties that can fluctuate over time. With increasing distance from the base years 1997 and 2001, we assume that uncertainties grow at a linear rate, thus covering all possibilities of structural change over time. To this end, we corrected prices in the GTAP data for 2001 for inflation back to 1997 using average, economy-wide producer price indices (PPI; OECD Statistics, 2008) for OECD EU, Non-EU OECD and four major Non-OECD trading partners, of which the average was used to estimate PPI data for the rest of the world. We then define and regress the absolute difference between the deflated table of 2001 and the original 1997 GTAP table as the RSDs of the 1997 data (Figure 4, ~6000 data points).

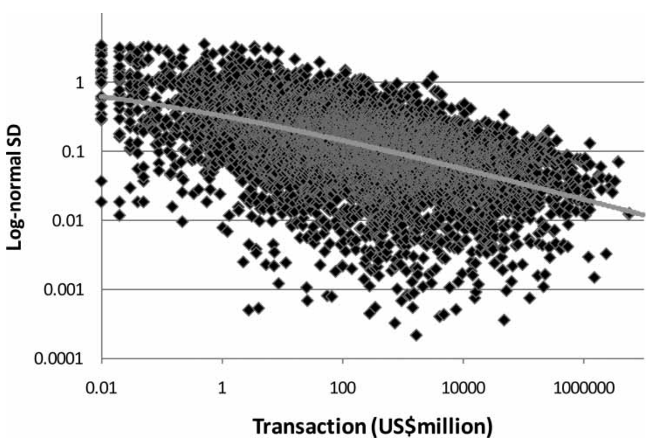
Apart from the fact that these PPIs are averaged over the whole economy and are not specific to individual sectors, we further account for the uncertainty of the deflators through variations of PPI data between countries (OECD Statistics, 2008). For each year between 1992 and 2004 we derived the average PPI of 15 EU countries as well as its SD across sectors. We then calculated the average SD over all years and, based on the results, use an RSD of 10% as a general estimate of uncertainty for all years and world regions.

We apply these regressions to the GTAP use data, but as we assume diagonal supply in the non-UK regions, we apply errors on supply elements calculated from gross output summing over use table transactions and final demand, i.e. for sector i,

$$d\mathbf{T}_{i,n+i} = \sqrt{\sum_{j=1:n} \left(d\mathrm{t}_{n+t,j}\right)^2 + \left(dy_{n+i}\right)^2}$$

where t_{ij} is the typical element of the transaction matrix $\mathbf{T}$ in supply/use format, n refers to number of sectors, and y_i is the typical element of $\mathbf{y}$, the final demand vector. We emphasize that no co-production can occur under the basic homogeneity assumption, and hence that all off-diagonal errors of the supply matrix are zero.

FIGURE 4. Distribution of differences (expressed as standard deviation) between original (year 1997) and deflated (year 2001) GTAP input–output data, representing structural change over a four year period.

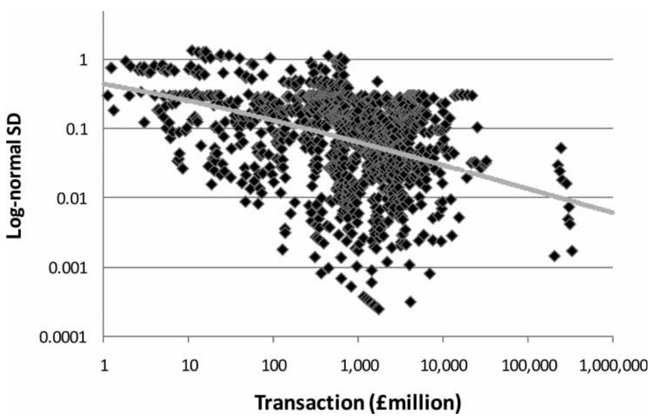


Note: $r_x = 1.144x^{-0.233}$, $R^2 = 0.237$.

Fifth, CO_2 emissions data for the three world regions were compiled from IEA data (IEA, 2006). No documentation of uncertainty could be found for this data set. However, the IEA data are in principle compiled in the same way as national GHG inventories and therefore the regression formula for the UK CO_2 data was used (see caption of Figure 2). Another challenge in terms of uncertainty is posed by the mismatch of numbers of sectors in the UK tables (123 sectors) and GTAP tables (30 sectors). This leads to a misallocation of foreign CO_2 intensities to UK sectors, and hence an over- or underestimation of CO_2 embodied in UK sectoral imports. For example, GTAP distinguishes only one transport sector (and hence provides only one CO_2 intensity for transport), whereas the UK table distinguishes five transport sectors with different CO_2 intensities. In principle it would be possible to alleviate this aggregation error for each aggregated GTAP sector block by using the variation of CO_2 intensities within a block of corresponding disaggregated UK sectors (Lenzen, 2001). However, it was not possible to carry out such a correction within the limited scope of project within which this work was funded.

Sixth, total imports into the UK by year and sector (UK Supply and Use Tables; ONS, 2007c) are associated with the SDs of UK input–output data as described above. In addition to this, further uncertainty is introduced through the fact that the sector totals for imports had to be divided into the three world regions used in the model. To accomplish this task, detailed trade data were compiled from various data sources and aggregated to the 123 sectors used in the input–output model (the exact procedure is described in detail in Wiedmann et al., 2008). When comparing the sum over the three ROW regions from this bottom-up compilation with the sector total for imports in the UK Supply tables for all years, an average difference of $\pm 20\%$ can be observed. This is likely to do with differences in classification, leading to differences in the way trade data were compiled (the SUTs follow SIC classification whereas the bottom-up data were classified by SITC classification). The sectoral differences identified by the comparison between the two trade data sets were regressed over the value of the transaction (1,200 data points; Figure 5).

FIGURE 5 Distribution of differences (expressed as standard deviation) between two data sets for UK imports, using different original data sources.



Note: $r_x = 1.787x^{-0.349}$, $R^2 = 0.142$.

2.2. Uncertainty of the MRIO Table

The UK input–output tables for the UK-MRIO model were obtained using the KRAS multi-proportional balancing algorithm (Lenzen *et al.*, 2009). KRAS forces the MRIO entries (‘vectorised’ as $\mathbf{p}$) to satisfy constraints $\mathbf{c}$ according to $\mathbf{Gp} = \mathbf{c}$, where $\mathbf{G}$ is a constraints coefficients matrix, and $\mathbf{c}$ holds the constraints values provided by the raw data. The balancing algorithm starts with an initial estimate $\mathbf{p}_0$ for $\mathbf{p}$, and arrives at a final solution $\mathbf{p}^{(\mathrm{final})}$, which is a vectorised form of the final MRIO table $\mathbf{T}$.

In most IO applications, and also in our work, the problem of constructing an IO table is underdetermined, that is there are many more IO table entries than raw data items to construct them. Hence, SDs of our MRIO cannot be readily assembled from the raw data. A first estimate of the vectorised SDs $\mathbf{dp}$ of the final MRIO $\mathbf{p}^{(\mathrm{final})}$ can be obtained by the shift that the $\mathbf{p}_0$ experience during the KRAS balancing run: $\mathbf{dp}^{(0)} = \mathbf{p}^{(\mathrm{final})} - \mathbf{p}_0$. However, this first estimate $\mathbf{dp}^{(0)}$ of the ‘true SDs’ $\mathbf{dp}$ would generally not comply with the SDs $\mathbf{dc}$ of the constraints, as constructed from the raw data $\mathbf{c}$ according to Section 2.1. In other words, $\mathbf{dp}^{(0)}$ would not propagate properly so that

$$\sqrt{\mathbf{G}\ \mathbf{dp}^{(0)}\ \#\ \mathbf{G}\ \mathbf{dp}^{(0)}} = \left\{\sqrt{\Sigma_j (g_{ij}\ \mathbf{d}p_j^{(0)})^2}\right\}_i = \mathbf{dc}^{(0)} \neq \mathbf{dc} = \sqrt{\mathbf{G}\ \mathbf{dp}\ \#\ \mathbf{G}\ \mathbf{dp}}$$

Where # denotes the element-wise product. A constraint-compliant vector $\mathbf{dp}$ can be determined by using a RAS-type balancing process as follows:

(1) Calculate a scalar $\mathrm{d}c_1/\mathrm{d}c^{(0)}{}_1$
(2) Adjust $\mathrm{d}p_j^{(1)} = \mathrm{d}p_j^{(0)} \times \mathrm{d}c_1/\mathrm{d}c^{(0)}{}_1\ \forall j$ where $g_{1j} \neq 0$, so that $\mathrm{d}c_1 = \sqrt{\sum_j \left(g_{1j}\mathbf{d}p_j^{(0)}\right)^2}$
(3) Calculate a scaler $\mathrm{d}c_2/\mathrm{d}c^{(0)}{}_2$.
(4) Adjust $\mathrm{d}p_j^{(1)} = \mathrm{d}p_j^{(0)} \times \mathrm{d}c_2/\mathrm{d}c^{(0)}{}_2\ \forall j$ where $g_{2\mathrm{j}} \neq 0$, so that $\mathrm{d}c_2 = \sqrt{\sum_j \left(g_{2j}\mathbf{d}p_j^{(0)}\right)^2}$
(5) And so on $\forall\ c_i$
(6) And so on for n iterations $\forall\ \mathrm{d}c_i\ /\ \mathrm{d}c^{(n)}{}_i$
(7) Exit at the nth step if $|\mathbf{dc} - \mathbf{dc}^{(n)}| \leq$ some small ε (else go to 1) and calculate $\mathrm{d}c_1/\mathrm{d}c^{(n+1)}{}_1$
(8) The $\mathbf{dp}^{(n)}$ are the solution for $\Delta\mathbf{p}$; they are the SDs $\Delta\mathbf{T}$ of the entries of the MRIO table $\mathbf{T}$.

The SDs $\mathbf{dq}$ of the direct, sectoral CO_2 intensities $\mathbf{q}$ are determined in a similar fashion.

2.3. Uncertainties of CO_2 Multipliers

CO_2 multipliers $\mathbf{m}$ are calculated from $\mathbf{T}$ according to $\mathbf{m} = \mathbf{q}[\mathbf{I} - \mathbf{T}\hat{\mathbf{x}}^{-1}]^{-1}$, where $\mathbf{q}$ are sectoral CO_2 intensities, $\hat{\mathbf{x}}$ is diagonalised sectoral gross output, and $\mathbf{I}$ is a suitable identity matrix. An exact analytical solution to the differential of $\mathbf{m}$ with regard to the elements of $\mathbf{T}$ does not exist, so that an approximation needs to be obtained using numerical techniques, notably Monte Carlo simulation. In our case, all SDs refer to the logarithms of variables, so that $\log_{10}\mathbf{x}^* = \log_{10}\mathbf{x} + \mathbf{dx}$, and hence we use Monte Carlo perturbations

$\mathbf{x}^* = 10\log_{10}^{\mathbf{x}+\boldsymbol{\delta}\mathbf{x}} = \mathbf{x} \# 10^{\boldsymbol{\delta}\mathbf{x}}$.[3] Here, the $\boldsymbol{\delta}\mathbf{x}$ denotes a vector of random variables distributed normally around mean 0 with SDs **dx**, and # denotes the element-wise product. Accordingly, we perturb entries of the MRIO table $\mathbf{T} \rightarrow \mathbf{T}^* = \mathbf{T} \# 10^{\boldsymbol{\delta}\mathbf{T}}$, gross output $\mathbf{x} \rightarrow \mathbf{x}^* = \mathbf{x} \# 10^{\boldsymbol{\delta}\mathbf{x}}$, and direct intensities $\mathbf{q} \rightarrow \mathbf{q}^* = \mathbf{q} \# 10^{\boldsymbol{\delta}\mathbf{q}}$.[4] We feed these into Monte Carlo simulations where we calculate perturbed CO_2 multipliers $\mathbf{m}^* = \mathbf{q}^*(\mathbf{I} - \mathbf{T}^*\hat{\mathbf{x}}^{*-1})^{-1} = \hat{\mathbf{x}}^{-1}\#10^{-\boldsymbol{\delta}\mathbf{x}})]^{-1}\ [\mathbf{I} - (\mathbf{T}\#10^{\boldsymbol{\delta}\mathbf{T}})(\hat{\mathbf{x}}^{-1}\#10^{-\boldsymbol{\delta}\mathbf{x}})]^{-1}$. We repeat this procedure a large number of times (5000 times per year), and compare perturbed and unperturbed multipliers in order to generate distributions of RSDs $\boldsymbol{\delta}\mathbf{m} = (\mathbf{m}^* - \mathbf{m})/\mathbf{m}$. The means of these distributions are then considered to be the actual RSDs **dm** of the CO_2 multipliers **m**.

In carrying out our Monte Carlo simulation, we have followed a conservative approach, which results in slightly higher RSD estimates, in two aspects:

1. Bullard and Sebald (1977; 1988) and Lenzen (2001) exclude 'infeasible' Monte Carlo runs with $|\boldsymbol{\delta}\mathbf{x}| \,/\, \mathbf{x} > 3\%$, since they assume that gross output is a macro quantity that is known with relatively high confidence. However, we do not restrict gross output at all, but instead include all Monte Carlo runs, thus including runs yielding multiplier deviations $\boldsymbol{\delta}\mathbf{m}$ and RSDs **dm** that are higher than those that would have resulted from output-restricted runs.
2. Perturbing the MRIO table **T** generally results in the perturbed table $\mathbf{T}^* = \mathbf{T}\#\ 10^{\boldsymbol{\delta}\mathbf{T}}$ being unbalanced, that is its row sums will not necessarily equal its column sums. In theory, row sums and column sums must balance (gross input = gross output). Applying a RAS-type balancing procedure in order to force $\mathbf{T}^*$ to conform with row and column balances would result in a reduced perturbance of $\mathbf{T}^*$, and hence in reduced multiplier deviations $\boldsymbol{\delta}\mathbf{m}$. We decided not to balance the perturbed table in order to reflect uncertainty of gross input and output, thus yielding higher-than-expected multiplier RSDs **dm**.

2.4. Uncertainty of Carbon Footprint Components

In the final stage of our calculations we combine the CO_2 multiplier SDs **dm** with the SDs **dy** of final demand **y** in order to obtain SDs for the carbon footprint components. Calculating these components involves simple additions and subtractions, so that the corresponding error propagation can be dealt with in the usual way by applying the 'square root of square sums' formula. Let $E_i = m_i y_i$ denote the emissions embodied in sector i's final demand y_i. Let $J = \{j\}$ be a set of sectors j, and $AE_j = \sum_{i \in J} E_i$ its aggregated embodied emissions. Let **dE** and **dAE** be the SDs of **E** and **AE**. Using the general error

---

[3] In fact, since we regress order-of-magnitude errors as $\delta x = \log_{10}(1 + r_x) = \log_{10}(1 + ax^b)$, we find that $x \# 10^{\delta x} = x + r_x$, which is nothing but perturbing matrix elements according to their relative standard deviations. However, the original report to the UK government used specifications of δx other than $\log_{10}$ forms, and the formulation used here covers these in all generality. It was only during the revision phase of this paper that new data could be located, and the $\log_{10}$ form emerged as the preferred specification for order-of-magnitude errors δx.

[4] Dietzenbacher (1995; 2005) reports on bias introduced into input–output multipliers during stochastic analyses, but concludes that in a 'practitioner's approach' where the transaction matrix is perturbed instead of the input–output coefficients matrix, such bias is negligibly small.

propagation for a function $f(x_i)$ as

$$\mathbf{d}f = \sqrt{\left(\frac{\partial f}{\partial x_i}\right)^2 \mathbf{d}x_i^2} \tag{3}$$

we find

$$\mathbf{d}E_i = \sqrt{m_i^2\mathbf{d}y_i^2 + \mathbf{d}m_i^2 y_i^2} \Leftrightarrow \frac{\mathbf{d}E_i}{E_i} = \sqrt{\frac{\mathbf{d}y_i^2}{y_i^2} + \frac{\mathbf{d}m_i^2}{m_i^2}} \tag{4}$$

and

$$AE_j = \sqrt{\sum_{i \in J} \mathbf{d}E_i^2} \tag{5}$$

3. RESULTS

3.1. Standard Deviations of UK Input–Output Table Entries

As expected, the procedure outlined in Section 2.2 yields RSDs of UK input–output tables (black circles in Figure 6 and 7) that group around the regressed RSDs for the raw data **c** (grey circles; there are two lines in the diagram because there are two specifications for dc, domestic and imports). This is because the constraints are sums of MRIO table elements, linked through the constraints matrix **G** via **Gp**=**c**. Therefore, the SDs of the constraints are square-rooted sums of squares of SDs of the table elements. In a typical sum, some SDs of input–output table elements are smaller than those of the constraints, some are larger. SDs of large table elements are below the

FIGURE 6. Calculated RSDs of UK input–output table entries, 1992.

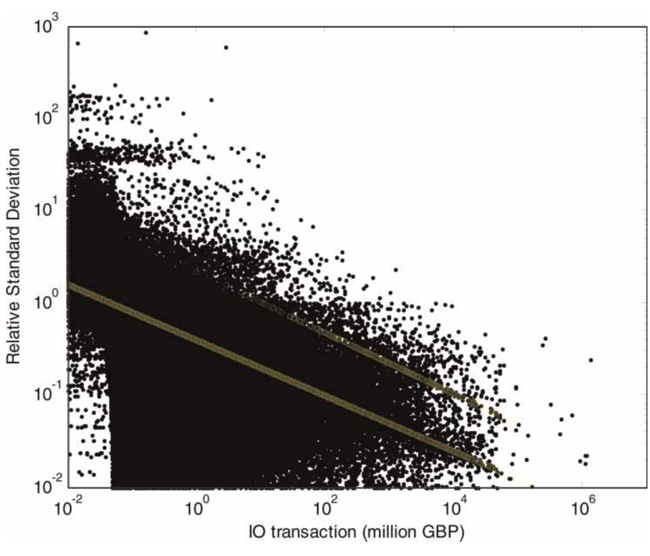
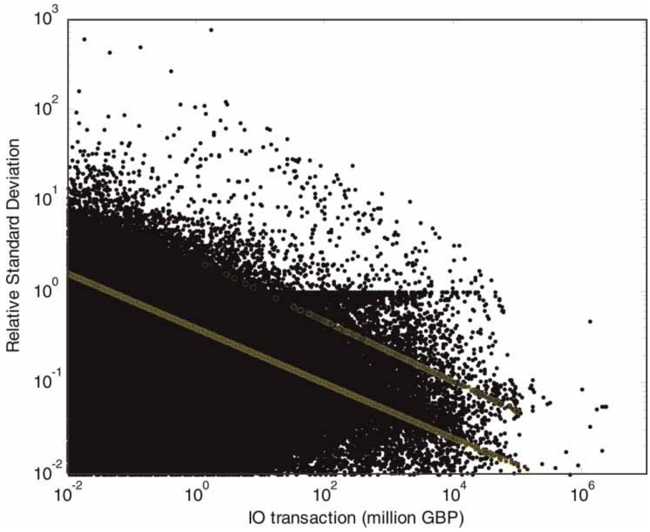


FIGURE 7. Calculated RSDs of UK input–output table entries, 2004.



SDs of raw data, because these elements are relatively stable under KRAS balancing, at the cost of small table elements with large specified RSDs that get ‘thrown around’ during the balancing procedure.

3.2. Carbon Footprint Components

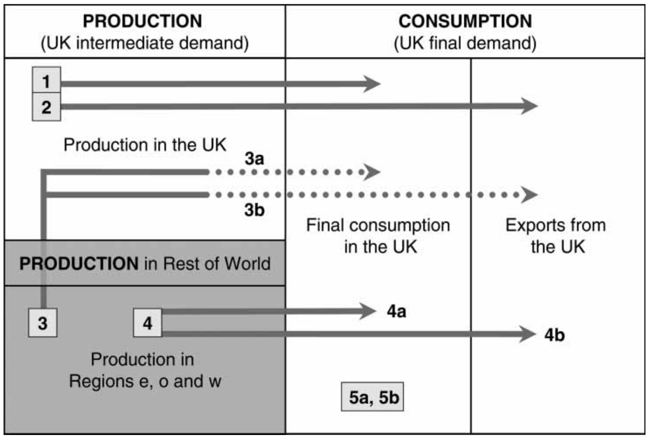
Before the final uncertainty results are presented, we repeat the nomenclature used in the original report to Defra (Wiedmann et al., 2008; see also Wiedmann et al., 2010). Figure 8 and the text below explain the various components of the UK’s carbon footprint (direct and embodied emissions) caused by UK production and consumption.

CE have increased notably between 1994 and 2004, mostly after 2000 (Figure 9). The RSD for total CE ranges from 3.0% in 1994 to 5.1% in 1999 and 2001. In order to test whether the highest CE emissions in 2001 emissions are statistically significantly higher than the lowest emissions in 1994, we calculate for each value E_{2001} within the distribution the probability that $E_{1994} < E_{2001}$. Each value E_{2001} occurs with a probability p_{2001}, which is simply calculated by enumerating a normal distribution $N(\mu_{2001},\sigma_{2001})$ specified by the 2001 emissions mean μ_{2001} and standard deviation σ_{2001}. The probability p_{1994} that 1994 emissions are below E_{2001} is expressed by the complementary error function $\mathrm{Erfc}(\mu_{1994},\sigma_{1994})$ evaluated at E_{2001}. Therefore, the total probability $p_{1994<2001}$ of 1994 emissions being below 2001 emissions is an integral over the probability of 2001 emissions p_{2001} conditional on p_{1994}, or

$$p_{1994<2001} = \int_{-\infty}^{\infty} p_{2001}(E)p_{1994}(E)\, dE = \int_{-\infty}^{\infty} \mathrm{N}(\mu_{2001,}\ \sigma_{1994}\ |E)\mathrm{Erfc}(\mu_{1994},\sigma_{1994}|E)dE$$

Setting $\mu_{1994} = 634$ Mt CO_2, $\sigma_{1994} = 19.0$ Mt CO_2, $\mu_{2001} = 732$ Mt CO_2, $\sigma_{2001} = 37.3$ Mt CO_2, we find $p_{1994<2001} = 0.93$. Hence, in a statistical sense, 2001 emissions are

FIGURE 8. Depiction of origin and attributed destination of GHG emissions caused by UK economic activity.



Note: see text and Wiedmann et al., 2010, for details. Region e = OECD Europe, Region o = OECD non-Europe, Region w = non-OECD countries.

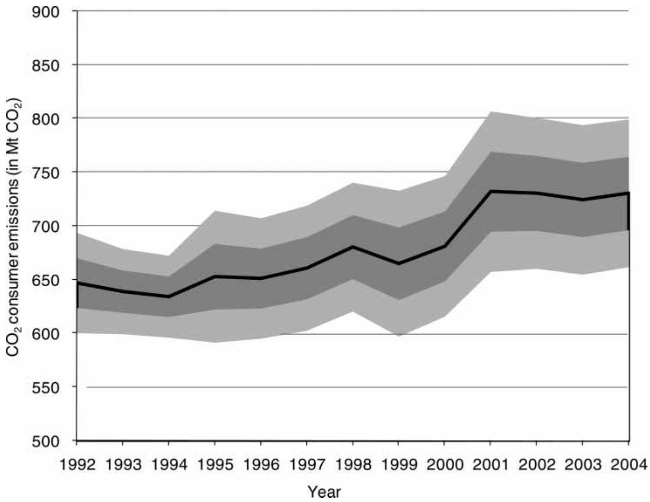
1 UK production emissions, including international aviation and shipping provided by UK operators, attributable to UK final consumption;
2 UK production emissions attributable to exports;
3a Imported emissions through intermediate consumption of UK industry attributable to UK final consumption;
3b Imported emissions through intermediate consumption of UK industry attributable to UK exports;
4a Imported emissions direct to final demand attributable to UK final consumption;
4b Imported emissions direct to final demand attributable to UK exports;
5a UK emissions generated by households not from private motoring (e.g. housing);
5b UK emissions generated by households from private motoring.
Producer Emissions (**PE**): 1 + 2 + 5a + 5b.
Consumer Emissions (**CE**): 1 + 3a + 4a + 5a + 5b.
Emissions Embodied in Imports (**EEI**): 3a + 3b + 4a + 4b.
Emissions Embodied in Exports (**EEE**): 2 + 3b + 4b.
Balance of Emissions Embodied in Trade (**BEET**): 2 − 3a − 4a.

significantly above 1994 emissions. Repeating the same calculation for 1992 and 2004 values (the beginning and end of the time series) results in $p_{1992<2004} = 0.89$, i.e. there is a probability of 89% that CE in 1992 were indeed lower than in 2004.

Over the time period 1994 to 2004 there is a tendency towards larger error margins. It is likely that this increase is due to the distance from the analytical input–output tables (ATs) for 1995. At the time of this work, the 1995 ATs were the only available information on the structure of imports to the UK. The import matrices for all other years were derived through KRAS balancing to match actual year totals. It is logical to assume that uncertainties increase when these totals move away from the 1995 base year totals, although this has not been proven in this study.

The RSD for CO_2 emissions embodied in UK trade is larger than for total consumer emissions because additional uncertainties for input–output and CO_2 data for the three world regions come into play (Table 1 and Figure 10). Generally, the greater the aggregation of the results, the smaller the RSD because the SDs of emission components are

FIGURE 9. UK consumer CO_2 emissions between 1992 and 2004.



Note: Uncertainty ranges are presented as confidence intervals of width σ (+/−1SD, 67% of all observations) and 2σ (+/−2SD, 95% of all observations).

added together via the 'square root of square sums' formula (see equation (4)). However, if a subtraction is involved in the calculation, as is the case for BEET (= 2-3a-4a), RSDs can become very large and even indefinite if the subtraction results in zero. Notwithstanding RSDs of around 7%, the increase in EEI is statistically significant, at least for the last four years compared with the first three years of the time series. In addition, EEI are not only significantly higher than EEE in all years from 1992 to 2004, but EEI also grow faster than EEE thus widening the gap between territorial (producer) emissions and consumer emissions. As a result, the trade balance BEET is increasingly negative.

4. DISCUSSION AND CONCLUSIONS

4.1. Discussion

Our analysis of uncertainties associated with the UK-MRIO model attempts to capture all possible variations of underlying data and calculation procedures: for the majority of data we take into account a number of stochastic variations. However, there may also be sources of systematic errors, e.g.:

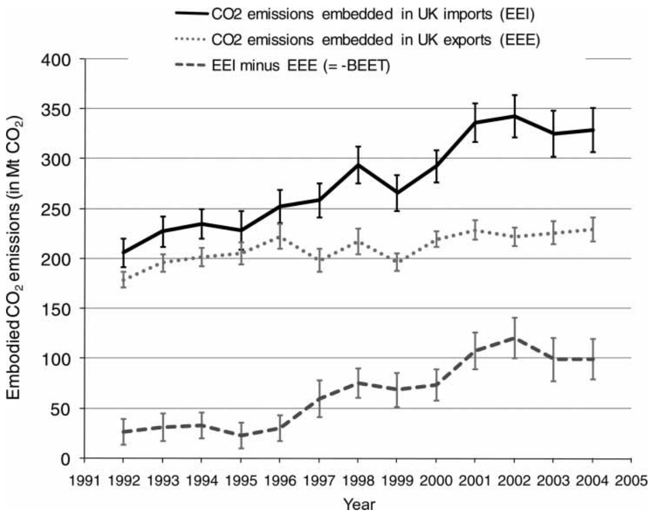
(1) structural change of foreign input–output data that cannot be captured systematically due to the lack of time series data;
(2) divergence in prices within sectoral foreign input–output data over time;
(3) systematic over- and underestimation of CO_2 intensities of foreign industries due to the mismatch of sectors in UK and foreign input–output and CO_2 data;
(4) change of import structure over time due to the lack of imports matrices (Analytical Tables) for the UK for years other than 1995; and

TABLE 1. CO_2 emissions embodied in UK trade and associated uncertainties (Mt CO_2).

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
EEI (3a + 3b + 4a + 4b)	206	227	235	228	253	259	294	266	293	336	343	325	329
Absolute SD of EEI	14.1	15.4	14.9	19.4	16.3	16.7	18.4	17.9	16.1	19.6	21.2	23.0	22.0
RSD of EEI	6.8%	6.8%	6.4%	8.5%	6.4%	6.4%	6.3%	6.7%	5.5%	5.8%	6.2%	7.1%	6.7%
EEE (2 + 3b + 4b)	179	196	202	205	222	199	218	197	219	229	222	226	230
Absolute SD of EEE	8.0	8.8	9.5	10.8	12.3	11.7	12.9	8.7	7.8	9.6	9.5	11.5	11.7
RSD of EEE	4.5%	4.5%	4.7%	5.3%	5.5%	5.9%	5.9%	4.4%	3.6%	4.2%	4.3%	5.1%	5.1%
BEET (2 − 3a − 4a)	−27	−31	−33	−23	−30	−60	−76	−69	−74	−108	−121	−99	−100
Absolute SD of BEET	12.7	13.7	12.6	12.6	12.7	18.4	14.7	16.8	15.5	18.4	20.2	21.5	20.4
RSD of BEET	47.1%	43.9%	38.2%	55.1%	41.8%	30.6%	19.4%	24.4%	21.1%	17.1%	16.8%	21.6%	20.5%

EEI/EEE = emissions embodied in imports/exports, BEET = balance of emissions embodied in trade, SD = standard deviation, RSD = relative standard deviation.

FIGURE 10. CO_2 emissions embodied in total UK imports (EEI), total UK exports (EEE) and the difference EEI-EEE (equal to −BEET) from 1992 to 2004.



Note: The error bars represent confidence intervals of width 1 σ (+/−1SD, 67% of all observations).

(5) choice of price conversion factors, e.g. Purchasing Power Parity (PPP) versus market exchange rate (MER), or choice of lead countries for PPI.

With regard to issue (5), results of calculations using PPP and MER can differ substantially because of crucial assumptions. Using MER means that a commodity produced in China would have 10 times more embodied emissions when sold in the UK for 10 times the price of consumption in China: The distorting factor is price differentials. Using PPP means that a commodity sold for the same price in China and in the UK would attract different embodied emissions: The distorting factor is the PPP adjustment to lower incomes in China. For further examples and elaborations on this issue see Nordhaus (2005), Li and Hewitt (2008) and Weber and Matthews (2008).

With regard to issue (3) and other systematic errors, we note that whilst it is in principle possible to investigate these systematic errors, this was beyond the scope of the work for Defra and this paper. We believe that we have left room for some systematic error by allowing our stochastic variations to be conservative, in the sense that they are rather over- than underestimated.

Finally, we note that, whilst aggregated results for CO_2 consumer emissions exhibit relatively low RSDs between 3.0% and 5.1%, RSDs at an individual sector level are generally higher. RSDs can be large especially for sectors with low CO_2 emissions. Results at a sectoral level (in Wiedmann et al., 2008) should therefore be interpreted with caution.

4.2. Conclusions

This is the first study that has undertaken a comprehensive uncertainty analysis of a global multi-region input–output model, using state-of-the-art techniques such as KRAS balancing under conflicting information, regression analysis for the estimation of raw data standard deviations, and Monte-Carlo simulation for determining uncertainties for

CO_2 multipliers and CO_2 emissions embodied in UK economic activity, as well as trade from and to the UK.

The usefulness and robustness of the techniques used in this work is demonstrated by the fact that they were successfully applied in commissioned work for a major government project, and that the funding agency chose to publish all data, methodology and results.

The results of our uncertainty analysis show with statistical significance that CO_2 emissions embodied in UK imports (EEI) were higher than those for exports (EEE) in all years from 1992 to 2004 and that EEI were growing faster than EEE thus widening the gap between territorial (producer) emissions and consumer emissions. Thus, against popular belief, the carbon footprint of the UK has been increasing rather than decreasing.

This finding may provide motivation for further investigations of other countries, perhaps a selection of technology-endowed developed countries and resource-endowed developing countries. Models such as the UK-MRIO can help shed light on the phenomenon of 'carbon leakage', that is developed countries reducing their reportable domestic emissions by outsourcing emissions-intensive production beyond their borders. Mounting evidence for the importance of international trade may ultimately serve to improve international reporting guidelines and emissions reduction agreements (Munksgaard and Pedersen, 2001; Peters, 2008b; Peters and Hertwich, 2008; Reinvang and Peters, 2008). Whilst current full producer responsibility may not be completely abandoned for full consumer responsibility, shared-responsibility principles may prove to be a compromise that is acceptable by a wide range of decision-makers (Lenzen et al., 2007; Andrew and Forgie, 2008).

Acknowledgements

This work was funded by the UK Department for Environment, Food and Rural Affairs (Defra) under Project Reference EV02033. We thank AEA Technology (Sarah Choudrie, Andrew Lelland, John Watterson), ONS (Ian Gazley, Sarah Green) and Defra (Jim Penman, Michele Pittini) for providing data as well as valuable information and feedback. Special thanks go to Dr Glen Peters from CICERO, Oslo for his explanation of GTAP data compilation and uncertainties, and also to Dr Christopher Weber from Carnegie Mellon University, Pittsburgh.

References

Andrew, R. and V. Forgie (2008) A Three-Perspective View of Greenhouse Gas Emission Responsibilities in New Zealand. *Ecological Economics*, 68, 194–204.

Beynon, M.J. and M. Munday (2008) Considering the Effects of Imprecision and Uncertainty in Ecological Foot-Print Estimation: An Approach in a Fuzzy Environment. *Ecological Economics*, 67, 373–383.

Bullard, C.W. and A.V. Sebald (1977) Effects of Parametric Uncertainty and Technological Change on Input–Output Models. *Review of Economics and Statistics*, 59, 75–81.

Bullard, C.W. and A.V. Sebald (1988) Monte Carlo Sensitivity Analysis of Input–Output Models. *The Review of Economics and Statistics*, 70, 708–712.

Defourny, J. and E. Thorbecke (1984) Structural Path Analysis and Multiplier Decomposition Within a Social Accounting Matrix Framework. *Economic Journal*, 94, 111–136.

Dietzenbacher, E. (1995) On the Bias of Multiplier Estimates. *Journal of Regional Science*, 35, 377–390.
Dietzenbacher, E. (2005) More on Multipliers. *Journal of Regional Science*, 45, 421–426.
Dimaranan, B.V. (2006) *Global Trade, Assistance, and Production: The GTAP 6 Data Base*. Center for Global Trade Analysis, Purdue University.
Evans, W.D. (1954) The Effect of Structural Matrix Errors on Interindustry Relations Estimates. *Econometrica*, 22, 461–480.
Goicoechea, A. and D.R. Hansen (1978) An Input–Output Model with Stochastic Parameters for Economic Analysis. *AIIE Transactions*, 10, 291–295.
Hanseman, D.J. (1982) Stochastic Input–Output Analysis: A Simulation Study. *Environment and Planning A*, 14, 1425–1435.
Hanseman, D.J. and E.F. Gustafson (1981) Stochastic Input–Output Analysis: A Comment. *Review of Economics and Statistics*, 63, 468–470.
Hondo, H. and S. Sakai (2001) Consistent Method for System Boundary Definition in LCA: An Application of Sensitivity Analysis. *Journal of Advanced Science*, 13, 491–494.
Hondo, H., S. Sakai and S. Tanno (2002) Sensitivity Analysis of Total CO_2 Emission Intensities Estimated Using an Input–Output Table. *Applied Energy*, 72, 689–704.
IEA (2006) *CO_2 Emissions from Fuel Combustion: 2006 Edition*. Paris, France International Energy Agency.
Jackson, R.W. and G.R. West (1989) Perspectives on Probabilistic Input–Output Analysis. In: R.E. Miller, K.R. Polenske and A.Z. Rose (eds.), *Frontiers of Input–Output Analysis*. New York, Oxford University Press, 209–221.
Jackson, J., S. Choudrie, G. Thistlethwaite, N. Passant, T. Murrells, J. Watterson, D. Mobbs, L. Cardenas, A. Thomson, A. Leech, Y. Li, A. Manning, C. Walker, N. Brophy, S. Sneddon, M. Pierce, J. Thomas and K. Brown (2009a) *UK Greenhouse Gas Inventory, 1990 to 2007 – ANNEX 7: Uncertainties*, AEAT/ENV/R/2764.
Jackson, J., S. Choudrie, G. Thistlethwaite, N. Passant, T. Murrells, J. Watterson, D. Mobbs, L. Cardenas, A. Thomson, A. Leech, Y. Li, A. Manning, C. Walker, N. Brophy, S. Sneddon, M. Pierce, J. Thomas and K. Brown (2009b) *UK Greenhouse Gas Inventory, 1990 to 2007 – Annual Report for Submission under the Framework Convention on Climate Change*, AEAT/ENV/R/2764.
Kop Jansen, P.S.M. (1994) Analysis of Multipliers in Stochastic Input–Output Systems. *Regional Science and Urban Economics*, 24, 55–74.
Kop Jansen, P.S.M. and T. Ten Raa (1990) The Choice of Model in the Construction of Input–Output Coefficients Matrices. *International Economic Review*, 31, 213–227.
Lenzen, M. (2001) Errors in Conventional and Input–Output-Based Life-Cycle Inventories. *Journal of Industrial Ecology*, 4, 127–148.
Lenzen, M. (2006) Structural Path Analysis of Ecosystem Networks. *Ecological Modelling*, 200, 334–342.
Lenzen, M., L.-L. Pade and J. Munksgaard (2004) CO_2 Multipliers in Multi-Region Input–Output Models. *Economic Systems Research*, 16, 391–412.
Lenzen, M., J. Murray, F. Sack and T. Wiedmann (2007) Shared Producer and Consumer Responsibility – Theory and Practice. *Ecological Economics*, 61, 27–42.
Lenzen, M., B. Gallego and R. Wood (2009) Matrix Balancing Under Conflicting Information. *Economic Systems Research*, 21, 23–44.
Li, Y. and C.N. Hewitt (2008) The Effect of Trade between China and the UK on National and Global Carbon Dioxide Emissions. *Energy Policy*, 36, 1907–1914.
McDougall, R.A. (2006) *Chapter 19 – Updating and Adjusting the Regional Input–Output Tables*. West Lafayette, IN, Purdue University, Center for Global Trade Analysis.
Munksgaard, J. and K.A. Pedersen (2001) CO_2 Accounts for Open Economies: Producer or Consumer Responsibility?. *Energy Policy*, 29, 327–334.
Nansai, K., S. Tohno and M. Kasahara (2001) Uncertainty of the Embodied CO_2 Emission Intensity and Reliability of Life Cycle Inventory Analysis by Input–Output Approach. *Energy and Resources*, 22, (published by the Japan Society of Energy and Resources, in Japanese).
Nordhaus, W.D. (2005) Alternative Measures of Output in Global Economic-Environmental Models: Purchasing Power Parity or Market Exchange Rates?. Meeting of IPCC Expert Meeting on Emission Scenarios, in Washington, DC.
OECD Statistics (2008) *Dataset 'PPPs and Exchange Rates'*, accessed on 5 February 2008.
ONS. (2007a) *Environmental Accounts: Greenhouse Gas Emissions for 93 industries, 2007 Edition*, (last update 19 July 2007).

ONS. (2007b) *Table: ABI Quality Measures, Annual Business Inquiry, Sections A-O Whole Economy – Standard Error By Group*. http://www.statistics.gov.uk/abi/quality_measures.asp. London, Office for National Statistics.

ONS. (2007c) *United Kingdom Input–Output Statistics*. London, Office for National Statistics (ONS).

Park, S. (1973) On Input–Output Multipliers with Errors in Input–Output Coefficients. *Journal of Economic Theory*, 6, 399–403.

Park, S.-H., M. Mohtadi and A. Kubursi (1981) Errors in Regional Nonsurvey Input–Output Models: Analytical and Simulation Results. *Journal of Regional Science*, 21, 321–339.

Peters, G. (2007a) Efficient Algorithms for Life Cycle Assessment, Input–Output Analysis, and Monte-Carlo Analysis. *International Journal of Life Cycle Assessment*, 12, 373–380.

Peters, G. (2007b) Opportunities and Challenges for Environmental MRIO Modeling: Illustrations with the GTAP Database. *16th International Input–Output Conference of the International Input–Output Association (IIOA)*. Istanbul, Turkey.

Peters, G. (2008a) Personal Communication. February 2008.

Peters, G. (2008b) From Production-Based to Consumption-Based National Emission Inventories. *Ecological Economics*, 65, 13–23.

Peters, G. and E.G. Hertwich (2006) Structural Analysis of International Trade: Environmental Impacts of Norway. *Economic Systems Research*, 18, 155–181.

Peters, G. and E.G. Hertwich (2008) CO_2 Embodied in International Trade with Implications for Global Climate Policy. *Environmental Science and Technology*, 42, 1401–1407.

Quandt, R.E. (1958) Probabilistic Errors in the Leontief System. *Naval Research Logistics Quarterly*, 5, 155–170.

Quandt, R.E. (1959) On the Solution of Probabilistic Leontief Systems. *Naval Research Logistics Quarterly*, 6, 295–305.

Reinvang, R. and G. Peters (2008) *Norwegian Consumption, Chinese Pollution – An Example of how OECD Imports Generate CO_2 Emissions in Developing Countries*. Trondheim, Norway, WWF Norway, WWF China Programme Office, Norwegian University of Science and Technology.

Rey, S.J., G.R. West and M.V. Janikas (2004) Uncertainty in Integrated Regional Models. *Economic Systems Research*, 16, 259–277.

Sakai, S., S. Tanno and H. Hondo (2000) Uncertainty Analysis for I/O Analysis Using Perturbation Method. *The Fourth International Conference on EcoBalance*. Tsukuba, Japan, 189–192.

Stevens, G. and G.A. Trainer (1980) Error Generation in Regional Input–Output Analysis and its Implications for Nonsurvey Models. In: S. Pleeter (ed.), *Economic Impact Analysis: Methodology and Applications*. Boston, MA, Martinus Nijhoff Publishing, 68–84.

Ten Raa, T. and M.F.J. Steel (1994) Revised Stochastic Analysis of an Input–Output Model. *Regional Science and Urban Economics*, 24, 361–371.

Ten Raa, T. and R. Van der Ploeg (1989) A Statistical Approach to the Problem of Negatives in Input–Output Analysis. *Economic Modelling*, 6, 2–19.

Treloar, G. (1997) Extracting Embodied Energy Paths from Input–Output Tables: Towards an Input–Output-Based Hybrid Energy Analysis Method. *Economic Systems Research*, 9, 375–391.

Weber, C.L. (2008) Uncertainties in Constructing Environmental Multiregional Input–Output Models. *International Input–Output Meeting on Managing the Environment*, 9–11 July 2008. Seville, Spain. http://www.upo.es/econ/IIOMME08/index.php

Weber, C.L. and H.S. Matthews (2007) Embodied Environmental Emissions in U.S. International Trade, 1997–2004. *Environmental Science & Technology*, 41, 4875–4881.

Weber, C.L. and S.H. Matthews (2008) Quantifying the Global and Distributional Aspects of an American Household Carbon Footprint. *Ecological Economics*, 66, 379–391.

West, G.R. (1986) A Stochastic Analysis of an Input–Output Model. *Econometrica*, 54, 363–374.

Wiedmann, T. (2009) A Review of Recent Multi-Region Input-Output Models used for Consumption-Based Emission and Resource Accounting. *Ecological Economics*, 69, 211–222.

Wiedmann, T., M. Lenzen, K. Turner and J. Barrett (2007) Examining the Global Environmental Impact of Regional Consumption Activities – Part 2: Review of Input–Output Models for the Assessment of Environmental Impacts Embodied in Trade. *Ecological Economics*, 61, 15–26.

Wiedmann, T., R. Wood, M. Lenzen, J. Minx, D. Guan and J. Barrett (2008) Development of an Embedded Carbon Emissions Indicator. Report by Stockholm Environment Institute at the University of York and Centre for Integrated Sustainability Analysis at the University of Sydney, London, UK Department for

Environment, Food and Rural Affairs. www.isa.org.usyd.edu.au/publications/documents/Defra_EmbeddedCarbon_Main.pdf

Wiedmann, T., R. Wood, J. Minx, M. Lenzen, D. Guan and R. Harris (2010) A Carbon Footprint Time Series of the UK – Results from a Multi-Region Input–Output Model. *Economic Systems Research*, 22, 19–42.

WWF Trade and Investment Programme and Norwegian University of Science and Technology Industrial Ecology Programme (2008) *EU Consumption, Global Pollution*. Gland, Switzerland, WWF International.

Yoshida, Y., H. Ishitani, R. Matsuhashi, Y. Kudoh, H. Okuma, K. Morita, A. Koike, O. Kobayashi and K. Yoshioka (2001) Evaluation of Uncertainty in LCA Based on Input–Output Analysis. *Technology*, 8, 141–148.

Yoshida, Y., H. Ishitani, R. Matsuhashi, Y. Kudoh, H. Okuma, K. Morita, A. Koike and O. Kobayashi (2002) Reliability of LCI Considering the Uncertainties of Energy Consumptions in Input–Output Analyses. *Applied Energy*, 73, 71–82.